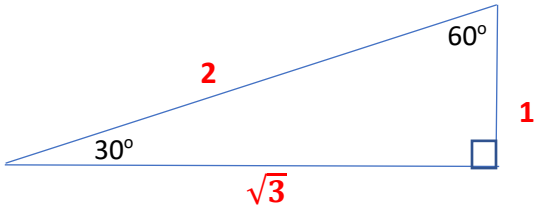
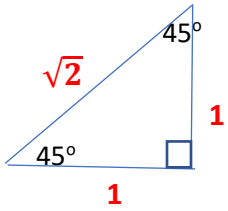
Mastering the Unit Circle for Trigonometry



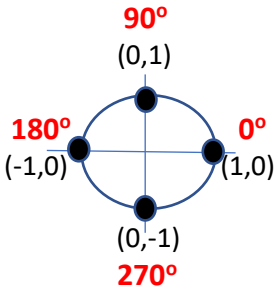
For a 30-60-90 Triangle, the "short leg" is opposite the 30° angle, the "long leg" is opposite the 60° angle, and "hypotenuse" is opposite the 90° angle. Set the "short leg" to 1. The hypotenuse always doubles the "short leg" in a 30-60-90 Triangle, which means the value will be 2. The "long leg" is $\sqrt{3}$ times bigger than the "short leg. That provides the following Trig Ratios:

- Sin 30° = $\frac{1}{2}$
- Cos 30° = $\frac{\sqrt{3}}{2}$
- Tan 30° = $\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$
- Sin 60° = $\frac{\sqrt{3}}{2}$
- Cos 60° = $\frac{1}{2}$
- Tan 60° = $\frac{\sqrt{3}}{1} = \sqrt{3}$



For a 45-45-90 Triangle, the two legs are equal in length, because for any triangle, when two angles are equal, the sides opposite those angles have the same lengths. Set each leg to 1. The hypotenuse is $\sqrt{2}$ times bigger than the legs. Set the hypotenuse to $\sqrt{2}$. That provides the following Trig Ratios:

- Sin 45° = $\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$
- Cos 45° = $\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$
- Tan 45° = $\frac{1}{1}$ = 1



For the Unit Circle, angles on the X and Y axes are 0°, 90°, 180°, and 270°. Determine the (x,y) coordinate at each point. The x value will represent Cos of the angle, and the y value will represent the Sin of the angle. The definition of Tan is $\frac{y}{x}$. That provides the following Trig Ratios:

- Sin 0° = y at 0° → 0
- Cos 0° = x at 0° → 1
- Tan 0° = y/x at 0° → 0/1 → 0
- Sin 90° = y at 90° → 1
- Cos 90° = x at 90° → 0
- Tan 90° = y/x at 90° → 1/0 → undefined
- Sin 180° = y at 180° → 0
- Cos 180° = x at 180° → -1
- Tan 180° = y/x at 180° → 0/-1 → 0
- Sin 270° = y at 270° → -1
- Cos 270° = x at 270° → 0
- Tan 270° = y/x at 270° → -1/0 → undefined

Reference Angles: allow one to calculated Trig Ratios in Quadrant II, Quadrant III, and Quadrant IV.

The definition of a Reference Angle is the absolute value of how many degrees the actual angle is from the x-axis.

- Quadrant I → The Reference Angle is the actual angle (because the x-axis is 0^o)
- Quadrant II → The Reference Angle is | Actual Angle – 180^o |
- Quadrant III → The Reference Angle is | Actual Angle – 180^o |
- Quadrant IV → The Reference Angle is | Actual Angle – 360^o |

Here are the Reference Angles all the way around the Unit Circle

- 30^o → Reference Angle = | $30^o – 0^o$ | = 30^o
- 45^o → Reference Angle = | $45^o – 0^o$ | = 45^o
- 60^o → Reference Angle = | $60^o – 0^o$ | = 60^o
- 120^o → Reference Angle = | $120^o – 180^o$ | = 60^o
- 135^o → Reference Angle = | $135^o – 180^o$ | = 45^o
- 150^o → Reference Angle = | $150^o – 180^o$ | = 30^o
- 210^o → Reference Angle = | $210^o – 180^o$ | = 30^o
- 225^o → Reference Angle = | $225^o – 180^o$ | = 45^o
- 240^o → Reference Angle = | $240^o – 180^o$ | = 60^o
- 300^o → Reference Angle = | $300^o – 360^o$ | = 60^o
- 315^o → Reference Angle = | $315^o – 360^o$ | = 45^o
- 330^o → Reference Angle = | $330^o – 360^o$ | = 30^o

The signs of the Trig Function depend on the quadrant:

- Cos is positive to the right (Quadrants I and IV) and negative to the left (Quadrants II and III)
- Sin is positive above (Quadrants I and II) and negative below (Quadrants III and IV)
- Tan is positive [+/+ and -/-] in (Quadrants I and II) and negative [+/- and -/+] in Quadrants II and IV

Here are the Trig Functions in Quadrants II, III, and IV (Using Reference Angles)

- Sin 120° → +Sin 60° = $\frac{\sqrt{3}}{2}$
- Cos 120° → -Cos 60° = -$\frac{1}{2}$
- Tan 120° → -Tan 60° = -$\sqrt{3}$
- Sin 135° → +Sin 45° = $\frac{\sqrt{2}}{2}$
- Cos 135° → -Cos 45° = -$\frac{\sqrt{2}}{2}$
- Tan 135° → -Tan 45° = -1
- Sin 150° → +Sin 30° = -$\frac{1}{2}$
- Cos 150° → -Cos 30° = -$\frac{\sqrt{3}}{2}$
- Tan 150° → -Tan 30° = -$\frac{\sqrt{3}}{3}$
- Sin 210° → -Sin 30° = -$\frac{1}{2}$
- Cos 210° → -Cos 30° = -$\frac{\sqrt{3}}{2}$
- Tan 210° → +Tan 30° = $\frac{\sqrt{3}}{3}$
- Sin 225° → -Sin 45° = -$\frac{\sqrt{2}}{2}$
- Cos 225° → -Cos 45° = -$\frac{\sqrt{2}}{2}$
- Tan 225° → +Tan 45° = 1
- Sin 240° → -Sin 60° = -$\frac{\sqrt{3}}{2}$
- Cos 240° → -Cos 60° = -$\frac{1}{2}$
- Tan 240° → +Tan 60° = $\sqrt{3}$
- Sin 300° → -Sin 60° = -$\frac{\sqrt{3}}{2}$
- Cos 300° → +Cos 60° = $\frac{1}{2}$
- Tan 300° = -Tan 60° = -$\sqrt{3}$
- Sin 315° → -Sin 45° = -$\frac{\sqrt{2}}{2}$
- Cos 315° → +Cos 45° = $\frac{\sqrt{2}}{2}$
- Tan 315° → -Tan 45° = -1
- Sin 330° → -Sin 30° = -$\frac{1}{2}$
- Cos 330° → +Cos 30° = $\frac{\sqrt{3}}{2}$
- Tan 330° → -Tan 30° = -$\frac{\sqrt{3}}{3}$