Answers to Odd-Numbered Exercises

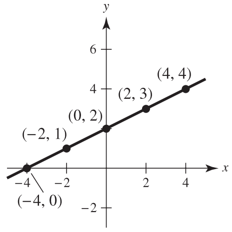
Chapter P

Section P.1 (page 8)

1. b **2.** d **3.** a **4.** c

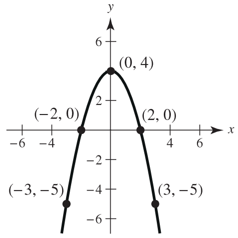
5. Answers will vary.

x	-4	-2	0	2	4
y	0	1	2	3	4



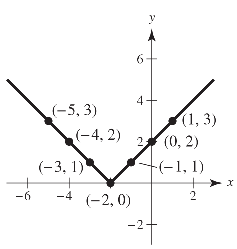
7. Answers will vary.

x	-3	-2	0	2	3
y	-5	0	4	0	-5



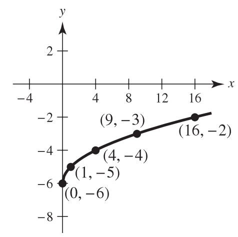
9. Answers will vary.

x	-5	-4	-3	-2	-1	0	1
y	3	2	1	0	1	2	3



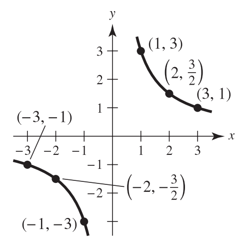
11. Answers will vary.

x	0	1	4	9	16
y	-6	-5	-4	-3	-2



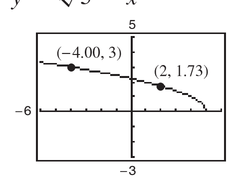
13. Answers will vary.

x	-3	-2	-1	0	1	2	3
y	-1	$-\frac{3}{2}$	-3	Undef.	3	$\frac{3}{2}$	1



15.
Xmin = -5
Xmax = 4
Xscl = 1
Ymin = -5
Ymax = 8
Yscl = 1

17. $y = \sqrt{5 - x}$



(a) $y \approx 1.73$ (b) $x = -4$

19. $(0, -5), \left(\frac{5}{2}, 0\right)$ **21.** $(0, -2), (-2, 0), (1, 0)$

23. $(0, 0), (4, 0), (-4, 0)$ **25.** $(4, 0)$ **27.** $(0, 0)$

29. Symmetric with respect to the y-axis

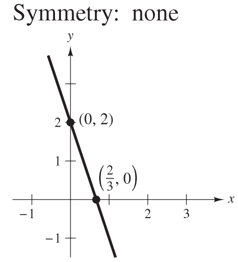
31. Symmetric with respect to the x-axis

33. Symmetric with respect to the origin **35.** No symmetry

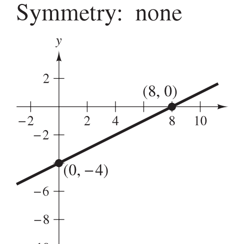
37. Symmetric with respect to the origin

39. Symmetric with respect to the y-axis

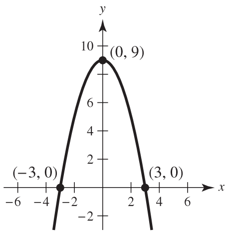
41. $y = 2 - 3x$
Symmetry: none



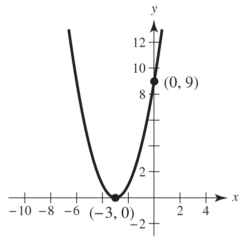
43. $y = \frac{1}{2}x - 4$
Symmetry: none



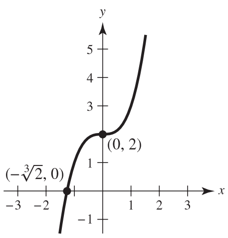
45. $y = 9 - x^2$
Symmetry: y-axis



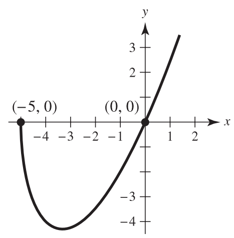
47. $y = (x + 3)^2$
Symmetry: none



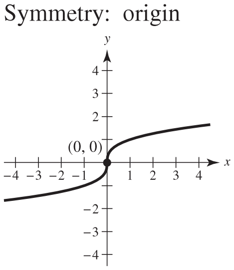
49. $y = x^3 + 2$
Symmetry: none



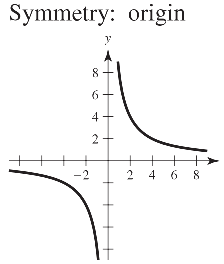
51. $y = x\sqrt{x + 5}$
Symmetry: none



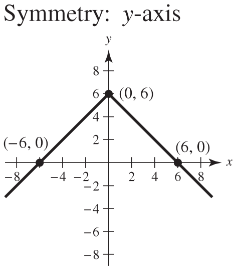
53. $x = y^3$
Symmetry: origin



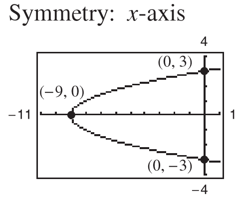
55. $y = 8/x$
Symmetry: origin



57. $y = 6 - |x|$
Symmetry: y-axis



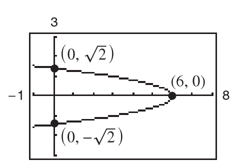
59. $y_1 = \sqrt{x + 9}$
$y_2 = -\sqrt{x + 9}$
Symmetry: x-axis



61. $y_1 = \sqrt{\dfrac{6 - x}{3}}$

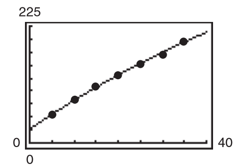
$y_2 = -\sqrt{\dfrac{6 - x}{3}}$

Symmetry: x-axis



63. $(3, 5)$ **65.** $(-1, 5), (2, 2)$ **67.** $(-1, -2), (2, 1)$
69. $(-1, -1), (0, 0), (1, 1)$
71. $(-1, -5), (0, -1), (2, 1)$ **73.** $(-2, 2), \left(-3, \sqrt{3}\right)$

75. (a) $y = -0.027t^2 + 5.73t + 26.9$
(b)



The model is a good fit for the data.

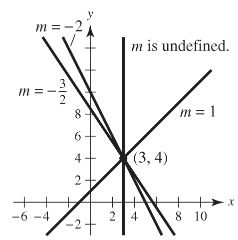
(c) 212.9

77. $x \approx 3133$ units **79.** $y = (x + 4)(x - 3)(x - 8)$
81. (a) Proof (b) Proof
83. False. $(4, -5)$ is not a point on the graph of $x = y^2 - 29$.
85. True **87.** $x^2 + (y - 4)^2 = 4$

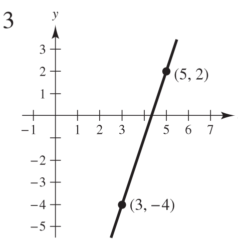
Section P.2 (page 16)

1. $m = 1$ **3.** $m = 0$ **5.** $m = -12$

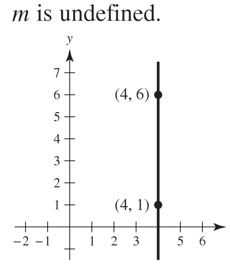
7.



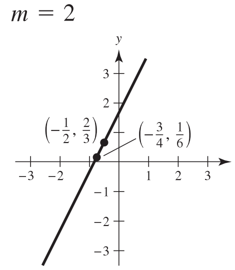
9. $m = 3$



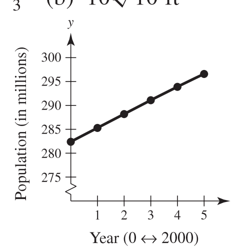
11. m is undefined.



13. $m = 2$

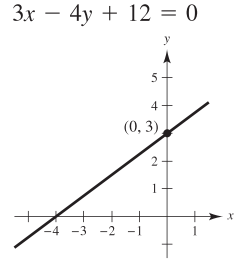


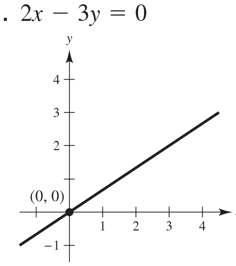
15. $(0, 2), (1, 2), (5, 2)$ **17.** $(0, 10), (2, 4), (3, 1)$
19. (a) $\frac{1}{3}$ (b) $10\sqrt{10}$ ft
21. (a)



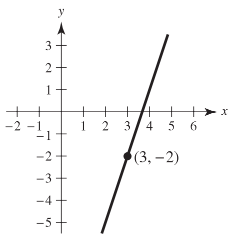
(b) Population increased least rapidly from 2004 to 2005.

23. $m = 4, (0, -3)$
25. $m = -\frac{1}{5}, (0, 4)$ **27.** m is undefined, no y-intercept
29. $3x - 4y + 12 = 0$ **31.** $2x - 3y = 0$

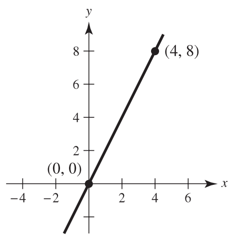




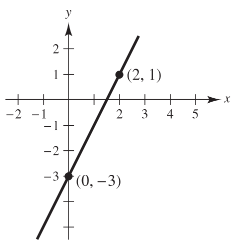
33. $3x - y - 11 = 0$



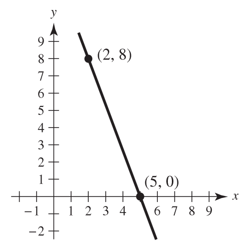
35. $2x - y = 0$



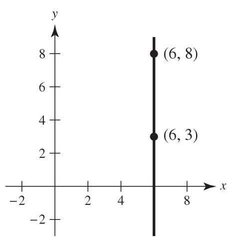
37. $2x - y - 3 = 0$



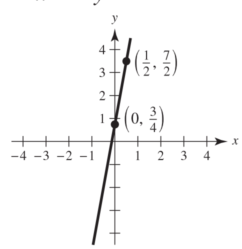
39. $8x + 3y - 40 = 0$



41. $x - 6 = 0$

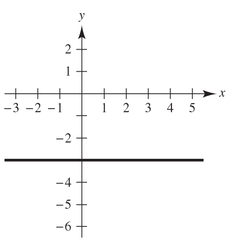


43. $22x - 4y + 3 = 0$

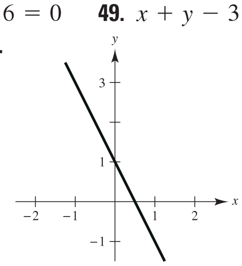


45. $x - 3 = 0$ **47.** $3x + 2y - 6 = 0$ **49.** $x + y - 3 = 0$

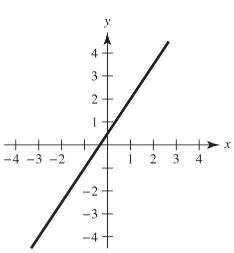
51.



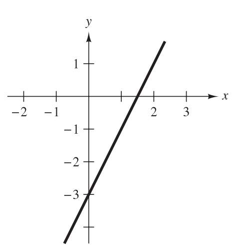
53.



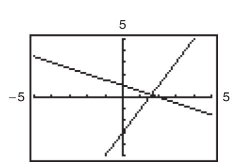
55.



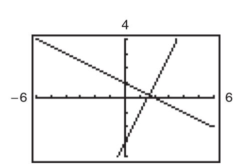
57.



59. (a)



(b)



The lines in (a) do not appear perpendicular, but they do in (b) because a square setting is used. The lines are perpendicular.

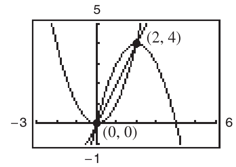
61. (a) $x + 7 = 0$ (b) $y + 2 = 0$

63. (a) $2x - y - 3 = 0$ (b) $x + 2y - 4 = 0$

65. (a) $40x - 24y - 9 = 0$ (b) $24x + 40y - 53 = 0$

67. $V = 250t - 150$ **69.** $V = -1600t + 30{,}000$

71. $y = 2x$



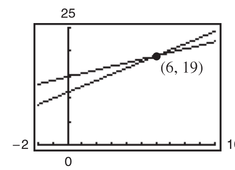
73. Not collinear, because $m_1 \neq m_2$

75. $\left(0, \dfrac{-a^2 + b^2 + c^2}{2c}\right)$ **77.** $\left(b, \dfrac{a^2 - b^2}{c}\right)$

79. $5F - 9C - 160 = 0$; $72°\text{F} \approx 22.2°\text{C}$

81. (a) $W_1 = 14.50 + 0.75x$, $W_2 = 11.20 + 1.30x$

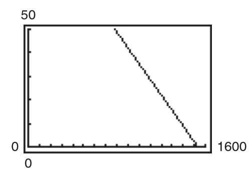
(b)



(c) When six units are produced, the wage for both options is \$19.00 per hour. Choose option 1 if you think you will produce less than six units and choose option 2 if you think you will produce more than six units.

83. (a) $x = (1530 - p)/15$

(b)



(c) 49 units

45 units

85. $12y + 5x - 169 = 0$ **87.** 2 **89.** $(5\sqrt{2})/2$ **91.** $2\sqrt{2}$

93. Proof **95.** Proof **97.** Proof **99.** True

Section P.3 (page 27)

1. (a) Domain of f: $[-4, 4]$; Range of f: $[-3, 5]$
Domain of g: $[-3, 3]$; Range of g: $[-4, 4]$
(b) $f(-2) = -1$; $g(3) = -4$
(c) $x = -1$ (d) $x = 1$ (e) $x = -1$, $x = 1$, and $x = 2$

3. (a) -4 (b) -25 (c) $7b - 4$ (d) $7x - 11$

5. (a) 5 (b) 0 (c) 1 (d) $4 + 2t - t^2$

7. (a) 1 (b) 0 (c) $-\frac{1}{2}$ **9.** $3x^2 + 3x\,\Delta x + (\Delta x)^2$, $\Delta x \neq 0$

11. $\left(\sqrt{x-1} - x + 1\right)/[(x-2)(x-1)]$
$= -1/\left[\sqrt{x-1}\left(1 + \sqrt{x-1}\right)\right]$, $x \neq 2$

13. Domain: $(-\infty, \infty)$; Range: $[0, \infty)$

15. Domain: $[0, \infty)$; Range: $[0, \infty)$

17. Domain: All real numbers t such that $t \neq 4n + 2$, where n is an integer; Range: $(-\infty, -1] \cup [1, \infty)$

19. Domain: $(-\infty, 0) \cup (0, \infty)$; Range: $(-\infty, 0) \cup (0, \infty)$

21. Domain: $[0, 1]$

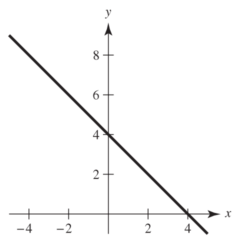
23. Domain: All real numbers x such that $x \neq 2n\pi$, where n is an integer

25. Domain: $(-\infty, -3) \cup (-3, \infty)$

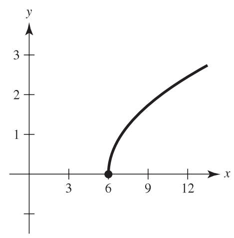
27. (a) -1 (b) 2 (c) 6 (d) $2t^2 + 4$
Domain: $(-\infty, \infty)$; Range: $(-\infty, 1) \cup [2, \infty)$

29. (a) 4 (b) 0 (c) -2 (d) $-b^2$
Domain: $(-\infty, \infty)$; Range: $(-\infty, 0] \cup [1, \infty)$

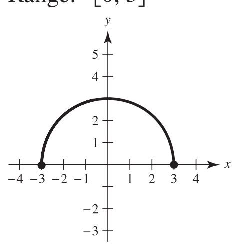
31. $f(x) = 4 - x$
Domain: $(-\infty, \infty)$
Range: $(-\infty, \infty)$



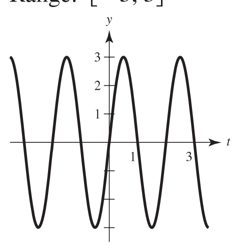
33. $h(x) = \sqrt{x - 6}$
Domain: $[6, \infty)$
Range: $[0, \infty)$



35. $f(x) = \sqrt{9 - x^2}$
Domain: $[-3, 3]$
Range: $[0, 3]$



37. $g(t) = 3 \sin \pi t$
Domain: $(-\infty, \infty)$
Range: $[-3, 3]$



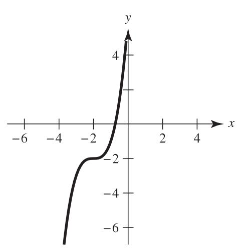
39. The student travels $\frac{1}{2}$ mile/minute during the first 4 minutes, is stationary for the next 2 minutes, and travels 1 mile/minute during the final 4 minutes.

41. y is not a function of x. **43.** y is a function of x.

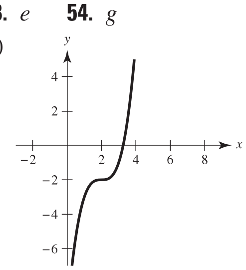
45. y is not a function of x. **47.** y is not a function of x.

49. *d* **50.** *b* **51.** *c* **52.** *a* **53.** *e* **54.** *g*

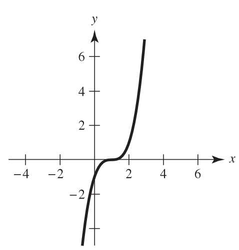
55. (a)



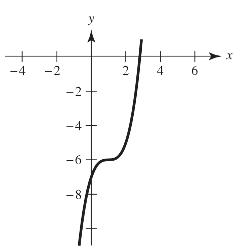
(b)



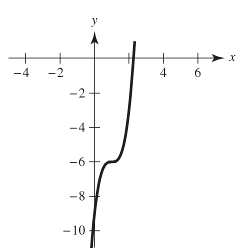
(c)



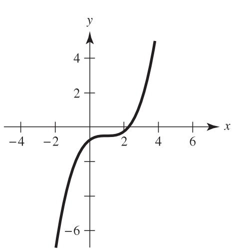
(d)



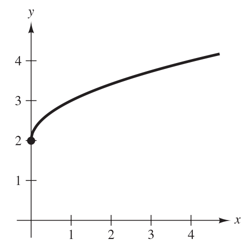
(e)



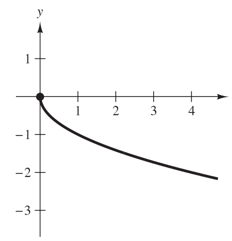
(f)



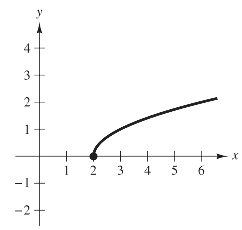
57. (a) Vertical translation



(b) Reflection about the x-axis



(c) Horizontal translation



59. (a) 0 (b) 0 (c) -1 (d) $\sqrt{15}$
(e) $\sqrt{x^2 - 1}$ (f) $x - 1\ (x \geq 0)$

61. $(f \circ g)(x) = x$; Domain: $[0, \infty)$
$(g \circ f)(x) = |x|$; Domain: $(-\infty, \infty)$
No, their domains are different.

63. $(f \circ g)(x) = 3/(x^2 - 1)$; Domain: $(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$
$(g \circ f)(x) = (9/x^2) - 1$; Domain: $(-\infty, 0) \cup (0, \infty)$
No

65. (a) 4 (b) -2
(c) Undefined. The graph of g does not exist at $x = -5$.
(d) 3 (e) 2
(f) Undefined. The graph of f does not exist at $x = -4$.

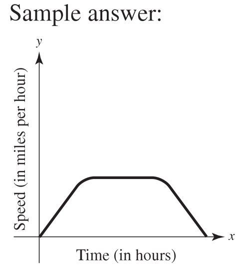
67. Answers will vary.
Example: $f(x) = \sqrt{x}$; $g(x) = x - 2$; $h(x) = 2x$

69. Even **71.** Odd **73.** (a) $\left(\frac{3}{2}, 4\right)$ (b) $\left(\frac{3}{2}, -4\right)$

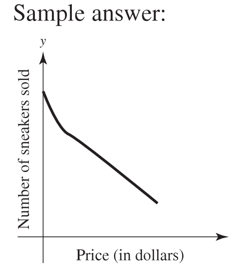
75. f is even. g is neither even nor odd. h is odd.

77. $f(x) = -5x - 6, -2 \leq x \leq 0$ **79.** $y = -\sqrt{-x}$

81. Answers will vary.
Sample answer:



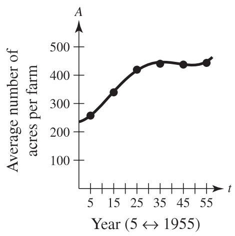
83. Answers will vary.
Sample answer:



85. $c = 25$

87. (a) $T(4) = 16°\text{C}, T(15) \approx 23°\text{C}$
(b) The changes in temperature occur 1 hour later.
(c) The temperatures are 1° lower.

89. (a)



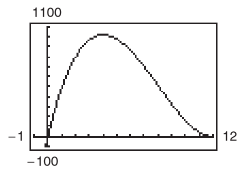
(b) $A(20) \approx 384$ acres/farm

91. $f(x) = |x| + |x - 2| = \begin{cases} 2x - 2, & \text{if } x \geq 2 \\ 2, & \text{if } 0 < x < 2 \\ -2x + 2, & \text{if } x \leq 0 \end{cases}$

93. Proof **95.** Proof

97. (a) $V(x) = x(24 - 2x)^2, 0 < x < 12$

(b)



$4 \times 16 \times 16$ cm

(c)

Height, *x*	*Length and Width*	*Volume*, *V*
1	$24 - 2(1)$	$1[24 - 2(1)]^2 = 484$
2	$24 - 2(2)$	$2[24 - 2(2)]^2 = 800$
3	$24 - 2(3)$	$3[24 - 2(3)]^2 = 972$
4	$24 - 2(4)$	$4[24 - 2(4)]^2 = 1024$
5	$24 - 2(5)$	$5[24 - 2(5)]^2 = 980$
6	$24 - 2(6)$	$6[24 - 2(6)]^2 = 864$

The dimensions of the box that yield a maximum volume are $4 \times 16 \times 16$ cm.

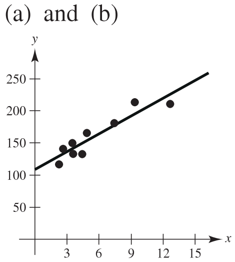
99. False. For example, if $f(x) = x^2$, then $f(-1) = f(1)$.

101. True **103.** Putnam Problem A1, 1988

Section P.4 (page 34)

1. Trigonometric **3.** No relationship

5. (a) and (b)

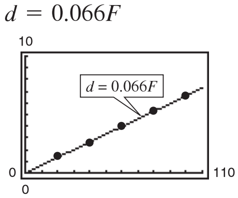


Approximately linear

(c) 136

7. (a) $d = 0.066F$

(b)

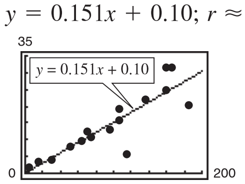


The model fits well.

(c) 3.63 cm

9. (a) $y = 0.151x + 0.10; r \approx 0.880$

(b)



(c) Greater per capita energy consumption by a country tends to correspond to greater per capita gross national product of the country. The four countries that differ most from the linear model are Venezuela, South Korea, Hong Kong, and the United Kingdom.

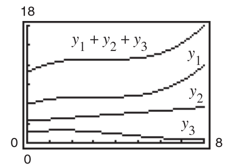
(d) $y = 0.155x + 0.22; r \approx 0.984$

11. (a) $y_1 = 0.04040t^3 - 0.3695t^2 + 1.123t + 5.88$

$y_2 = 0.264t + 3.35$

$y_3 = 0.01439t^3 - 0.1886t^2 + 0.476t + 1.59$

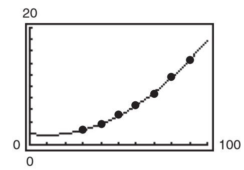
(b) $y_1 + y_2 + y_3 = 0.05479t^3 - 0.5581t^2 + 1.863t + 10.82$



About 47.5 cents/mi

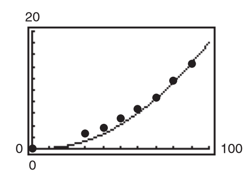
13. (a) $t = 0.002s^2 - 0.04s + 1.9$

(b)



(c) According to the model, the times required to attain speeds of less than 20 miles per hour are all about the same.

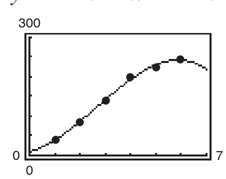
(d) $t = 0.002s^2 + 0.02s + 0.1$



(e) No. From the graph in part (b), you can see that the model from part (a) follows the data more closely than the model from part (d).

15. (a) $y = -1.806x^3 + 14.58x^2 + 16.4x + 10$

(b)

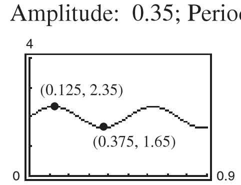


(c) 214 hp

17. (a) Yes. At time t there is one and only one displacement y.

(b) Amplitude: 0.35; Period: 0.5 (c) $y = 0.35 \sin(4\pi t) + 2$

(d)



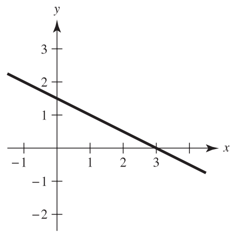
The model appears to fit the data well.

19. Answers will vary. **21.** Putnam Problem A2, 2004

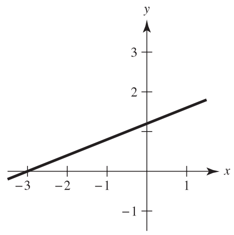
Review Exercises for Chapter P (page 37)

1. $\left(\frac{8}{5}, 0\right), (0, -8)$ **3.** $(3, 0), \left(0, \frac{3}{4}\right)$ **5.** y-axis symmetry

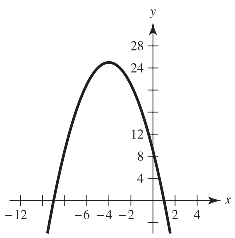
7.



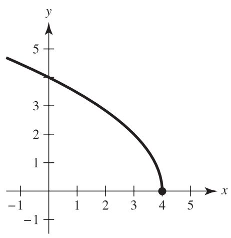
9.



11.



13.

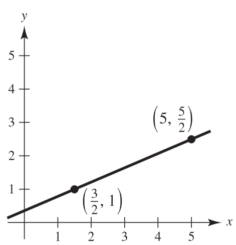


15.

Xmin = -5
Xmax = 5
Xscl = 1
Ymin = -30
Ymax = 10
Yscl = 5

17. $(-2, 3)$ **19.** $y = x^3 - 16x$

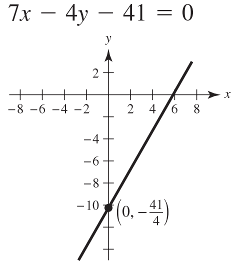
21.



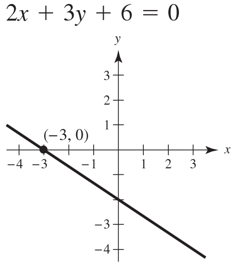
$m = \frac{3}{7}$

23. $t = \frac{1}{5}$

25. $y = \frac{7}{4}x - \frac{41}{4}$ or $7x - 4y - 41 = 0$



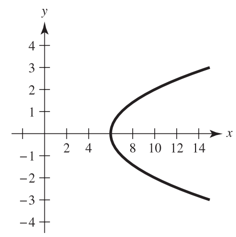
27. $y = -\frac{2}{3}x - 2$ or $2x + 3y + 6 = 0$



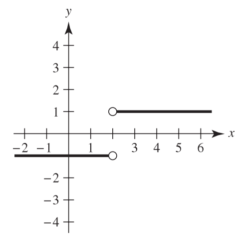
29. (a) $7x - 16y + 101 = 0$ (b) $5x - 3y + 30 = 0$
(c) $5x + 3y = 0$ (d) $x + 3 = 0$

31. $V = 12{,}500 - 850t$; \$9950

33. Not a function



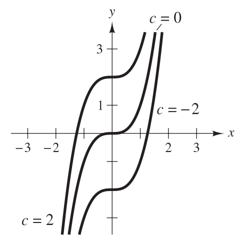
35. Function



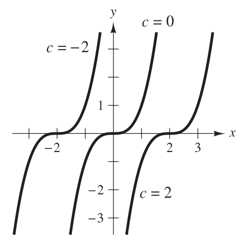
37. (a) Undefined (b) $-1/(1 + \Delta x), \Delta x \neq 0, -1$

39. (a) Domain: $[-6, 6]$; Range: $[0, 6]$
(b) Domain: $(-\infty, 5) \cup (5, \infty)$; Range: $(-\infty, 0) \cup (0, \infty)$
(c) Domain: $(-\infty, \infty)$; Range: $(-\infty, \infty)$

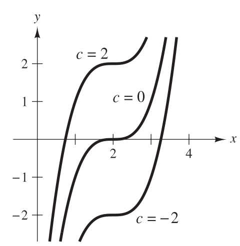
41. (a)



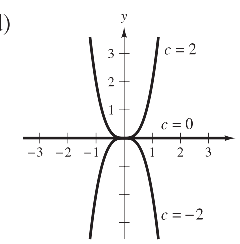
(b)



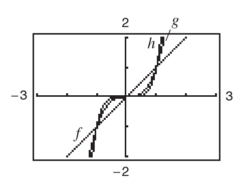
(c)

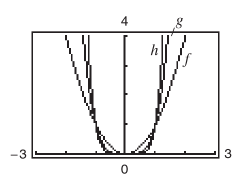


(d)



43. (a)

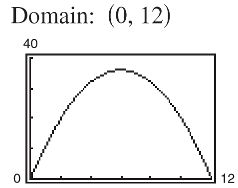




All the graphs pass through the origin. The graphs of the odd powers of x are symmetric with respect to the origin and the graphs of the even powers are symmetric with respect to the y-axis. As the powers increase, the graphs become flatter in the interval $-1 < x < 1$. Graphs of these equations with odd powers pass through Quadrants I and III. Graphs of these equations with even powers pass through Quadrants I and II.

(b) The graph of $y = x^7$ should pass through the origin and Quadrants I and III. It should be symmetric with respect to the origin and be fairly flat in the interval $(-1, 1)$. The graph of $y = x^8$ should pass through the origin and Quadrants I and II. It should be symmetric with respect to the y-axis and be fairly flat in the interval $(-1, 1)$.

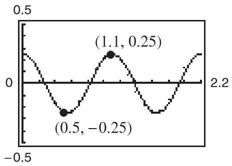
45. (a) $A = x(12 - x)$
(b) Domain: $(0, 12)$



(c) Maximum area: 36 in.^{2}; 6×6 in.

47. (a) Minimum degree: 3; Leading coefficient: negative
(b) Minimum degree: 4; Leading coefficient: positive
(c) Minimum degree: 2; Leading coefficient: negative
(d) Minimum degree: 5; Leading coefficient: positive

49. (a) Yes. For each time t there corresponds one and only one displacement y.
(b) Amplitude: 0.25; Period: 1.1 (c) $y \approx \frac{1}{4}\cos(5.7t)$
(d)

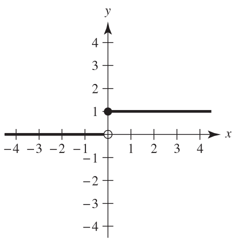


The model appears to fit the data.

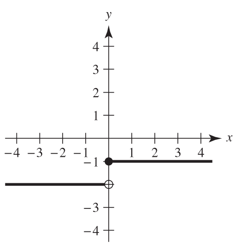
P.S. Problem Solving (page 39)

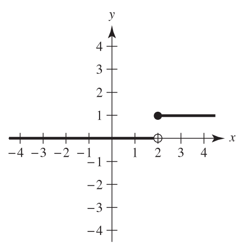
1. (a) Center: (3, 4); Radius: 5
(b) $y = -\frac{3}{4}x$ (c) $y = \frac{3}{4}x - \frac{9}{2}$ (d) $\left(3, -\frac{9}{4}\right)$

3.

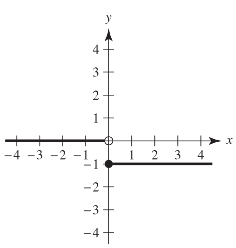


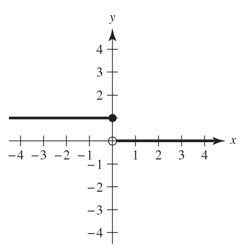
(a) $H(x) - 2 = \begin{cases} -1, & x \geq 0 \\ -2, & x < 0 \end{cases}$ (b) $H(x - 2) = \begin{cases} 1, & x \geq 2 \\ 0, & x < 2 \end{cases}$



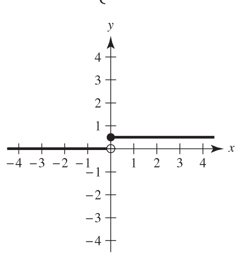


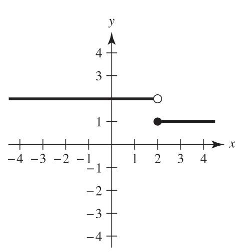
(c) $-H(x) = \begin{cases} -1, & x \geq 0 \\ 0, & x < 0 \end{cases}$ (d) $H(-x) = \begin{cases} 1, & x \leq 0 \\ 0, & x > 0 \end{cases}$



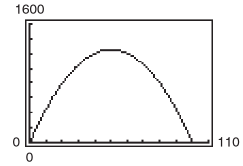


(e) $\frac{1}{2}H(x) = \begin{cases} \frac{1}{2}, & x \geq 0 \\ 0, & x < 0 \end{cases}$ (f) $-H(x - 2) + 2 = \begin{cases} 1, & x \geq 2 \\ 2, & x < 2 \end{cases}$





5. (a) $A(x) = x[(100 - x)/2]$; Domain: (0, 100)
(b)



Dimensions 50 m × 25 m yield maximum area of 1250 m^2.

(c) 50 m × 25 m; Area = 1250 m^2

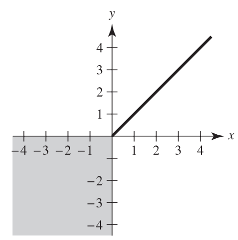
7. $T(x) = \left[2\sqrt{4 + x^2} + \sqrt{(3 - x)^2 + 1}\right]/4$

9. (a) 5, less (b) 3, greater (c) 4.1, less
(d) $4 + h$ (e) 4; Answers will vary.

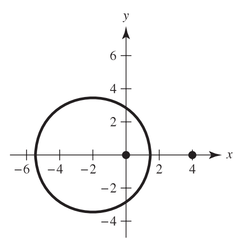
11. Using the definition of absolute value, you can rewrite the equation as

$$\begin{cases} 2y, & y > 0 \\ 0, & y \leq 0 \end{cases} = \begin{cases} 2x, & x > 0 \\ 0, & x \leq 0 \end{cases}.$$

For $x > 0$ and $y > 0$, you have $2y = 2x \rightarrow y = x$. For any $x \leq 0$, y is any $y \leq 0$. So, the graph of $y + |y| = x + |x|$ is as follows.

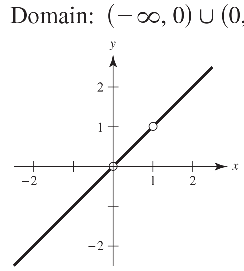


13. (a) $\left(x + \dfrac{4}{k - 1}\right)^2 + y^2 = \dfrac{16k}{(k - 1)^2}$
(b)



(c) As k becomes very large, $\dfrac{4}{k - 1} \rightarrow 0$ and $\dfrac{16k}{(k - 1)^2} \rightarrow 0$.
The center of the circle gets closer to (0, 0), and its radius approaches 0.

15. (a) Domain: $(-\infty, 1) \cup (1, \infty)$; Range: $(-\infty, 0) \cup (0, \infty)$
(b) $f(f(x)) = \dfrac{x - 1}{x}$
Domain: $(-\infty, 0) \cup (0, 1) \cup (1, \infty)$
(c) $f(f(f(x))) = x$
Domain: $(-\infty, 0) \cup (0, 1) \cup (1, \infty)$
(d)



The graph is not a line because there are holes at $x = 0$ and $x = 1$.

Chapter 1

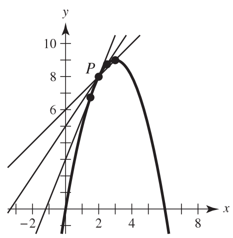
Section 1.1 (page 47)

1. Precalculus: 300 ft

3. Calculus: Slope of the tangent line at $x = 2$ is 0.16.

5. (a) Precalculus: 10 square units (b) Calculus: 5 square units

7. (a)



(b) $1; \frac{3}{2}; \frac{5}{2}$

(c) 2. Use points closer to P.

9. (a) Area $\approx$ 10.417; Area $\approx$ 9.145 (b) Use more rectangles.

11. (a) 5.66 (b) 6.11 (c) Increase the number of line segments.

Section 1.2 (page 54)

1.

x	3.9	3.99	3.999	4.001	4.01	4.1
$f(x)$	0.2041	0.2004	0.2000	0.2000	0.1996	0.1961

$$\lim_{x\to 4} \frac{x-4}{x^2-3x-4} \approx 0.2000 \left(\text{Actual limit is } \frac{1}{5}.\right)$$

3.

x	−0.1	−0.01	−0.001	0.001	0.01	0.1
$f(x)$	0.2050	0.2042	0.2041	0.2041	0.2040	0.2033

$$\lim_{x\to 0} \frac{\sqrt{x+6}-\sqrt{6}}{x} \approx 0.2041 \left(\text{Actual limit is } \frac{1}{2\sqrt{6}}.\right)$$

5.

x	2.9	2.99	2.999
$f(x)$	−0.0641	−0.0627	−0.0625

x	3.001	3.01	3.1
$f(x)$	−0.0625	−0.0623	−0.0610

$$\lim_{x\to 3} \frac{[1/(x+1)]-(1/4)}{x-3} \approx -0.0625 \left(\text{Actual limit is } -\frac{1}{16}.\right)$$

7.

x	−0.1	−0.01	−0.001
$f(x)$	0.9983	0.99998	1.0000

x	0.001	0.01	0.1
$f(x)$	1.0000	0.99998	0.9983

$$\lim_{x\to 0} \frac{\sin x}{x} \approx 1.0000 \text{ (Actual limit is 1.)}$$

9.

x	0.9	0.99	0.999	1.001	1.01	1.1
$f(x)$	0.2564	0.2506	0.2501	0.2499	0.2494	0.2439

$$\lim_{x\to 1} \frac{x-2}{x^2+x-6} \approx 0.2500 \left(\text{Actual limit is } \frac{1}{4}.\right)$$

11.

x	0.9	0.99	0.999	1.001	1.01	1.1
$f(x)$	0.7340	0.6733	0.6673	0.6660	0.6600	0.6015

$$\lim_{x\to 1} \frac{x^4-1}{x^6-1} \approx 0.6666 \left(\text{Actual limit is } \frac{2}{3}.\right)$$

13.

x	−0.1	−0.01	−0.001	0.001	0.01	0.1
$f(x)$	1.9867	1.9999	2.0000	2.0000	1.9999	1.9867

$$\lim_{x\to 0} \frac{\sin 2x}{x} \approx 2.0000 \text{ (Actual limit is 2.)}$$

15. 1 **17.** 2

19. Limit does not exist. The function approaches 1 from the right side of 2 but it approaches −1 from the left side of 2.

21. 0

23. Limit does not exist. The function oscillates between 1 and −1 as x approaches 0.

25. (a) 2

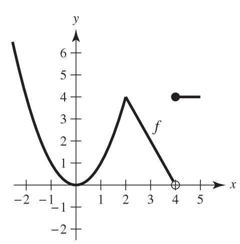
(b) Limit does not exist. The function approaches 1 from the right side of 1 but it approaches 3.5 from the left side of 1.

(c) Value does not exist. The function is undefined at $x = 4$.

(d) 2

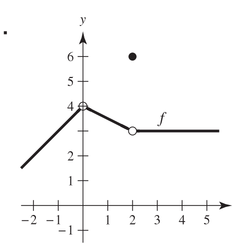
27. $\lim_{x\to c} f(x)$ exists for all points on the graph except where $c = -3$.

29.

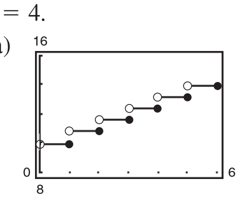


$\lim_{x\to c} f(x)$ exists for all points on the graph except where $c = 4$.

31.



33. (a)



(b)

t	3	3.3	3.4	3.5	3.6	3.7	4
C	11.57	12.36	12.36	12.36	12.36	12.36	12.36

$\lim_{t \to 3.5} C(t) = 12.36$

(c)

t	2	2.5	2.9	3	3.1	3.5	4
C	10.78	11.57	11.57	11.57	12.36	12.36	12.36

The limit does not exist because the limits from the right and left are not equal.

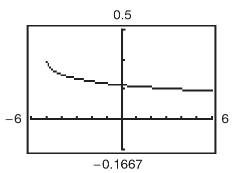
35. $\delta = 0.4$ **37.** $\delta = \frac{1}{11} \approx 0.091$

39. $L = 8$. Let $\delta = 0.01/3 \approx 0.0033$.

41. $L = 1$. Let $\delta = 0.01/5 = 0.002$.

43. 6 **45.** -3 **47.** 3 **49.** 0 **51.** 10 **53.** 2 **55.** 4

57.

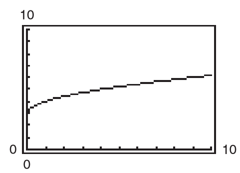


$\lim_{x \to 4} f(x) = \frac{1}{6}$

Domain: $[-5, 4) \cup (4, \infty)$

The graph has a hole at $x = 4$.

59.



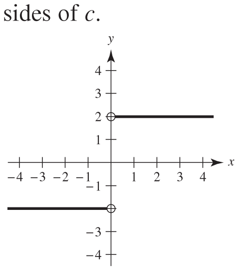
$\lim_{x \to 9} f(x) = 6$

Domain: $[0, 9) \cup (9, \infty)$

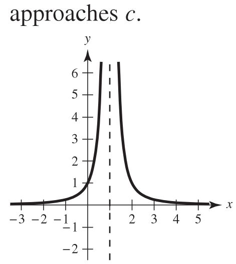
The graph has a hole at $x = 9$.

61. Answers will vary. Sample answer: As x approaches 8 from either side, $f(x)$ becomes arbitrarily close to 25.

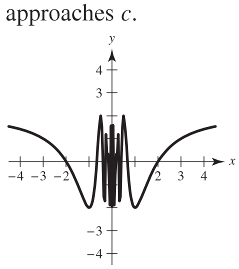
63. (i) The values of f approach different numbers as x approaches c from different sides of c.



(ii) The values of f increase or decrease without bound as x approaches c.



(iii) The values of f oscillate between two fixed numbers as x approaches c.



65. (a) $r = \frac{3}{\pi} \approx 0.9549$ cm

(b) $\frac{5.5}{2\pi} \le r \le \frac{6.5}{2\pi}$, or approximately $0.8754 < r < 1.0345$

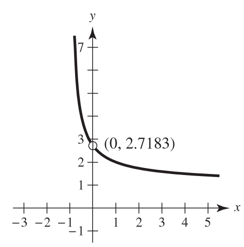
(c) $\lim_{r \to 3/\pi} 2\pi r = 6$; $\varepsilon = 0.5$; $\delta \approx 0.0796$

67.

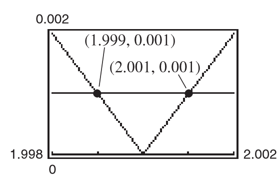
x	-0.001	-0.0001	-0.00001
$f(x)$	2.7196	2.7184	2.7183

x	0.00001	0.0001	0.001
$f(x)$	2.7183	2.7181	2.7169

$\lim_{x \to 0} f(x) \approx 2.7183$



69.



$\delta = 0.001$
(1.999, 2.001)

71. False. The existence or nonexistence of $f(x)$ at $x = c$ has no bearing on the existence of the limit of $f(x)$ as $x \to c$.

73. False. See Exercise 17.

75. Yes. As x approaches 0.25 from either side, $\sqrt{x}$ becomes arbitrarily close to 0.5.

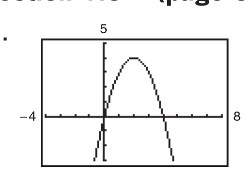
77. $\lim_{x \to 0} \frac{\sin nx}{x} = n$

79–81. Proofs **83.** Answers will vary.

85. Putnam Problem B1, 1986

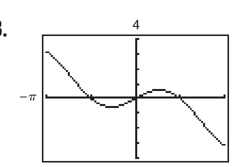
Section 1.3 (page 67)

1.



(a) 0 (b) -5

3.



(a) 0 (b) About 0.52 or $\pi/6$

5. 8 **7.** -1 **9.** 0 **11.** 7 **13.** 2 **15.** 1

17. 1/2 **19.** 1/5 **21.** 7 **23.** (a) 4 (b) 64 (c) 64

25. (a) 3 (b) 2 (c) 2 **27.** 1 **29.** 1/2 **31.** 1

33. 1/2 **35.** -1 **37.** (a) 10 (b) 5 (c) 6 (d) 3/2

39. (a) 64 (b) 2 (c) 12 (d) 8

41. (a) -1 (b) -2

$g(x) = \frac{x^2 - x}{x}$ and $f(x) = x - 1$ agree except at $x = 0$.

43. (a) 2 (b) 0

$g(x) = \dfrac{x^3 - x}{x - 1}$ and $f(x) = x^2 + x$ agree except at $x = 1$.

45. -2

$f(x) = \dfrac{x^2 - 1}{x + 1}$ and $g(x) = x - 1$ agree except at $x = -1$.

47. 12

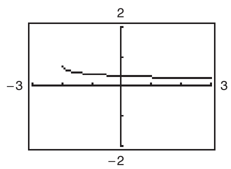
$f(x) = \dfrac{x^3 - 8}{x - 2}$ and $g(x) = x^2 + 2x + 4$ agree except at $x = 2$.

49. -1 **51.** $1/8$ **53.** $5/6$ **55.** $1/6$ **57.** $\sqrt{5}/10$

59. $-1/9$ **61.** 2 **63.** $2x - 2$

65. $1/5$ **67.** 0 **69.** 0 **71.** 0 **73.** 1 **75.** $3/2$

77.



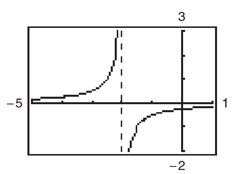
The graph has a hole at $x = 0$.

Answers will vary. Example:

x	-0.1	-0.01	-0.001	0.001	0.01	0.1
$f(x)$	0.358	0.354	0.354	0.354	0.353	0.349

$\lim_{x \to 0} \dfrac{\sqrt{x + 2} - \sqrt{2}}{x} \approx 0.354 \left(\text{Actual limit is } \dfrac{1}{2\sqrt{2}} = \dfrac{\sqrt{2}}{4}.\right)$

79.



The graph has a hole at $x = 0$.

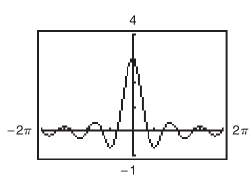
Answers will vary. Example:

x	-0.1	-0.01	-0.001
$f(x)$	-0.263	-0.251	-0.250

x	0.001	0.01	0.1
$f(x)$	-0.250	-0.249	-0.238

$\lim_{x \to 0} \dfrac{[1/(2 + x)] - (1/2)}{x} \approx -0.250 \left(\text{Actual limit is } -\dfrac{1}{4}.\right)$

81.



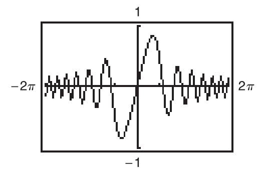
The graph has a hole at $t = 0$.

Answers will vary. Example:

t	-0.1	-0.01	0	0.01	0.1
$f(t)$	2.96	2.9996	?	2.9996	2.96

$\lim_{t \to 0} \dfrac{\sin 3t}{t} = 3$

83.



The graph has a hole at $x = 0$.

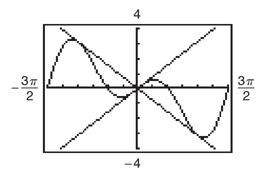
Answers will vary. Example:

x	-0.1	-0.01	-0.001	0	0.001	0.01	0.1
$f(x)$	-0.1	-0.01	-0.001	?	0.001	0.01	0.1

$\lim_{x \to 0} \dfrac{\sin x^2}{x} = 0$

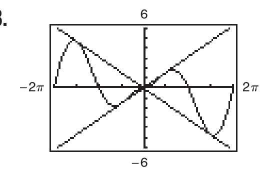
85. 3 **87.** $-1/(x + 3)^2$ **89.** 4

91.



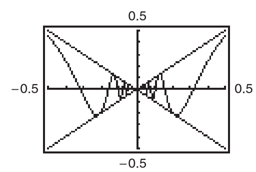
0

93.



0

95.



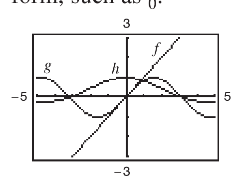
0

The graph has a hole at $x = 0$.

97. f and g agree at all but one point if c is a real number such that $f(x) = g(x)$ for all $x \neq c$.

99. An indeterminate form is obtained when evaluating a limit using direct substitution produces a meaningless fractional form, such as $\frac{0}{0}$.

101.



The magnitudes of $f(x)$ and $g(x)$ are approximately equal when x is close to 0. Therefore, their ratio is approximately 1.

103. -64 ft/sec (speed = 64 ft/sec) **105.** -29.4 m/sec

107. Let $f(x) = 1/x$ and $g(x) = -1/x$.

$\lim_{x \to 0} f(x)$ and $\lim_{x \to 0} g(x)$ do not exist. However,

$\lim_{x \to 0} [f(x) + g(x)] = \lim_{x \to 0} \left[\dfrac{1}{x} + \left(-\dfrac{1}{x}\right)\right] = \lim_{x \to 0} 0 = 0$

and therefore does exist.

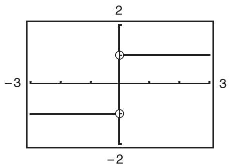
109–113. Proofs

115. Let $f(x) = \begin{cases} 4, & \text{if } x \geq 0 \\ -4, & \text{if } x < 0 \end{cases}$

$\lim_{x \to 0} |f(x)| = \lim_{x \to 0} 4 = 4$

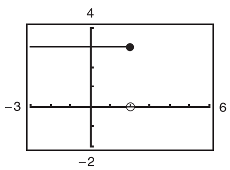
$\lim_{x \to 0} f(x)$ does not exist because for $x < 0$, $f(x) = -4$ and for $x \geq 0$, $f(x) = 4$.

117. False. The limit does not exist because the function approaches 1 from the right side of 0 and approaches -1 from the left side of 0. (See graph below.)



119. True.

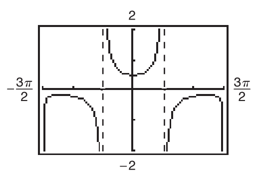
121. False. The limit does not exist because $f(x)$ approaches 3 from the left side of 2 and approaches 0 from the right side of 2. (See graph below.)



123. Proof

125. (a) All $x \neq 0, \dfrac{\pi}{2} + n\pi$

(b)



The domain is not obvious. The hole at $x = 0$ is not apparent from the graph.

(c) $\dfrac{1}{2}$ (d) $\dfrac{1}{2}$

127. The graphing utility was not set in *radian* mode.

Section 1.4 (page 78)

1. (a) 3 (b) 3 (c) 3; $f(x)$ is continuous on $(-\infty, \infty)$.

3. (a) 0 (b) 0 (c) 0; Discontinuity at $x = 3$

5. (a) -3 (b) 3 (c) Limit does not exist. Discontinuity at $x = 2$

7. $\frac{1}{16}$ **9.** $\frac{1}{10}$

11. Limit does not exist. The function decreases without bound as x approaches -3 from the left.

13. -1 **15.** $-1/x^2$ **17.** $5/2$ **19.** 2

21. Limit does not exist. The function decreases without bound as x approaches π from the left and increases without bound as x approaches π from the right.

23. 8

25. Limit does not exist. The function approaches 5 from the left side of 3 but approaches 6 from the right side of 3.

27. Discontinuous at $x = -2$ and $x = 2$

29. Discontinuous at every integer

31. Continuous on $[-7, 7]$ **33.** Continuous on $[-1, 4]$

35. Nonremovable discontinuity at $x = 0$

37. Continuous for all real x

39. Nonremovable discontinuities at $x = -2$ and $x = 2$

41. Continuous for all real x

43. Nonremovable discontinuity at $x = 1$
Removable discontinuity at $x = 0$

45. Continuous for all real x

47. Removable discontinuity at $x = -2$
Nonremovable discontinuity at $x = 5$

49. Nonremovable discontinuity at $x = -7$

51. Continuous for all real x

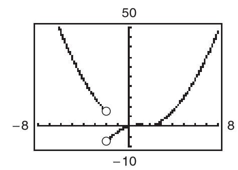
53. Nonremovable discontinuity at $x = 2$

55. Continuous for all real x

57. Nonremovable discontinuities at integer multiples of $\pi/2$

59. Nonremovable discontinuities at each integer

61.



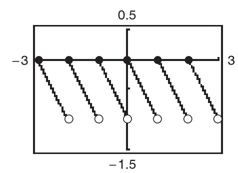
$\lim_{x\to 0^+} f(x) = 0$
$\lim_{x\to 0^-} f(x) = 0$
Discontinuity at $x = -2$

63. $a = 7$ **65.** $a = 2$ **67.** $a = -1,\ b = 1$

69. Continuous for all real x

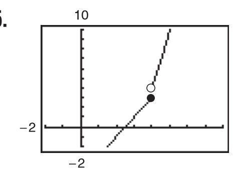
71. Nonremovable discontinuities at $x = 1$ and $x = -1$

73.



Nonremovable discontinuity at each integer

75.

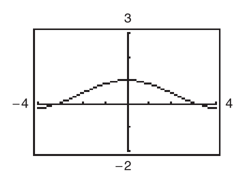


Nonremovable discontinuity at $x = 4$

77. Continuous on $(-\infty, \infty)$

79. Continuous on the open intervals . . . $(-6, -2)$, $(-2, 2)$, $(2, 6)$, ...

81.



The graph has a hole at $x = 0$. The graph appears to be continuous, but the function is not continuous on $[-4, 4]$. It is not obvious from the graph that the function has a discontinuity at $x = 0$.

83. Because $f(x)$ is continuous on the interval $[1, 2]$ and $f(1) = 37/12$ and $f(2) = -8/3$, by the Intermediate Value Theorem there exists a real number c in $[1, 2]$ such that $f(c) = 0$.

85. Because $f(x)$ is continuous on the interval $[0, \pi]$ and $f(0) = -3$ and $f(\pi) \approx 8.87$, by the Intermediate Value Theorem there exists a real number c in $[0, \pi]$ such that $f(c) = 0$.

87. 0.68, 0.6823 **89.** 0.56, 0.5636

91. $f(3) = 11$ **93.** $f(2) = 4$

95. (a) The limit does not exist at $x = c$.
(b) The function is not defined at $x = c$.
(c) The limit exists, but it is not equal to the value of the function at $x = c$.
(d) The limit does not exist at $x = c$.

97. If f and g are continuous for all real x, then so is $f + g$ (Theorem 1.11, part 2). However, f/g might not be continuous if $g(x) = 0$. For example, let $f(x) = x$ and $g(x) = x^2 - 1$. Then f and g are continuous for all real x, but f/g is not continuous at $x = \pm 1$.

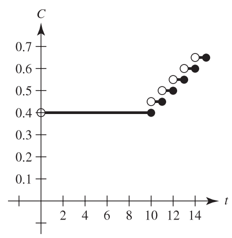
99. True

101. False. A rational function can be written as $P(x)/Q(x)$ where P and Q are polynomials of degree m and n, respectively. It can have, at most, n discontinuities.

103. $\lim_{t\to 4^-} f(t) \approx 28$; $\lim_{t\to 4^+} f(t) \approx 56$

At the end of day 3, the amount of chlorine in the pool is about 28 oz. At the beginning of day 4, the amount of chlorine in the pool is about 56 oz.

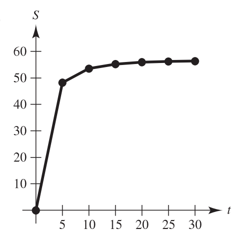
105. $C = \begin{cases} 0.40, & 0 < t \le 10 \\ 0.40 + 0.05[\![t - 9]\!], & t > 10,\ t \text{ is not an integer} \\ 0.40 + 0.05(t - 10), & t > 10,\ t \text{ is an integer} \end{cases}$



There is a nonremovable discontinuity at each integer greater than or equal to 10.

107–109. Proofs **111.** Answers will vary.

113. (a)

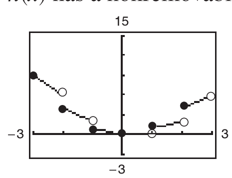


(b) There appears to be a limiting speed, and a possible cause is air resistance.

115. $c = \left(-1 \pm \sqrt{5}\right)/2$

117. Domain: $[-c^2, 0) \cup (0, \infty)$; Let $f(0) = 1/(2c)$

119. $h(x)$ has a nonremovable discontinuity at every integer except 0.



121. Putnam Problem B2, 1988

Section 1.5 (page 88)

1. $\lim_{x\to 4^+} \dfrac{1}{x-4} = \infty$, $\lim_{x\to 4^-} \dfrac{1}{x-4} = -\infty$

3. $\lim_{x\to 4^+} \dfrac{1}{(x-4)^2} = \infty$, $\lim_{x\to 4^-} \dfrac{1}{(x-4)^2} = \infty$

5. $\lim_{x\to -2^+} 2\left|\dfrac{x}{x^2-4}\right| = \infty$, $\lim_{x\to -2^-} 2\left|\dfrac{x}{x^2-4}\right| = \infty$

7. $\lim_{x\to -2^+} \tan(\pi x/4) = -\infty$, $\lim_{x\to -2^-} \tan(\pi x/4) = \infty$

9.

x	-3.5	-3.1	-3.01	-3.001
$f(x)$	0.31	1.64	16.6	167

x	-2.999	-2.99	-2.9	-2.5
$f(x)$	-167	-16.7	-1.69	-0.36

$\lim_{x\to -3^+} f(x) = -\infty$ $\lim_{x\to -3^-} f(x) = \infty$

11.

x	-3.5	-3.1	-3.01	-3.001
$f(x)$	3.8	16	151	1501

x	-2.999	-2.99	-2.9	-2.5
$f(x)$	-1499	-149	-14	-2.3

$\lim_{x\to -3^+} f(x) = -\infty$ $\lim_{x\to -3^-} f(x) = \infty$

13. $x = 0$ **15.** $x = \pm 2$ **17.** No vertical asymptote

19. $x = 2$, $x = -1$ **21.** $t = 0$ **23.** $x = -2, x = 1$

25. No vertical asymptote **27.** No vertical asymptote

29. $x = \frac{1}{2} + n$, n is an integer.

31. $t = n\pi$, n is a nonzero integer.

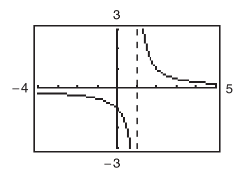
33. Removable discontinuity at $x = -1$

35. Vertical asymptote at $x = -1$ **37.** ∞ **39.** ∞

41. ∞ **43.** $-\frac{1}{5}$ **45.** $\frac{1}{2}$ **47.** $-\infty$ **49.** ∞ **51.** 0

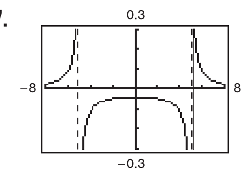
53. Limit does not exist.

55.



$\lim_{x\to 1^+} f(x) = \infty$

57.

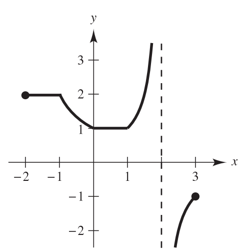


$\lim_{x\to 5^-} f(x) = -\infty$

59. Answers will vary.

61. Answers will vary. Example: $f(x) = \dfrac{x-3}{x^2 - 4x - 12}$

63.



65. ∞

67. (a) $\frac{1}{3}(200\pi)$ ft/sec

(b) 200π ft/sec

(c) $\lim_{\theta\to(\pi/2)^-} [50\pi \sec^2 \theta] = \infty$

69. (a) Domain: $x > 25$

(b)

x	30	40	50	60
y	150	66.667	50	42.857

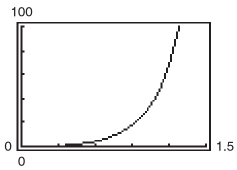
(c) $\lim_{x\to 25^+} \dfrac{25x}{x-25} = \infty$

As x gets closer and closer to 25 mi/h, y becomes larger and larger.

71. (a) $A = 50 \tan \theta - 50\theta$; Domain: $(0, \pi/2)$

(b)

θ	0.3	0.6	0.9	1.2	1.5
$f(\theta)$	0.47	4.21	18.0	68.6	630.1



(c) $\lim_{\theta \to \pi/2^-} A = \infty$

73. False; let $f(x) = (x^2 - 1)/(x - 1)$

75. False; let $f(x) = \tan x$

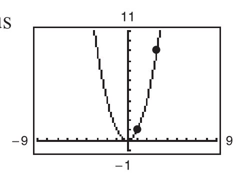
77. Let $f(x) = \dfrac{1}{x^2}$ and $g(x) = \dfrac{1}{x^4}$, and let $c = 0$. $\lim_{x \to 0} \dfrac{1}{x^2} = \infty$ and $\lim_{x \to 0} \dfrac{1}{x^4} = \infty$, but $\lim_{x \to 0} \left(\dfrac{1}{x^2} - \dfrac{1}{x^4}\right) = \lim_{x \to 0} \left(\dfrac{x^2 - 1}{x^4}\right) = -\infty \neq 0$.

79. Given $\lim_{x \to c} f(x) = \infty$, let $g(x) = 1$. Then $\lim_{x \to c} \dfrac{g(x)}{f(x)} = 0$ by Theorem 1.15.

81. Answers will vary.

Review Exercises for Chapter 1 (page 91)

1. Calculus



Estimate: 8.3

3.

x	−0.1	−0.01	−0.001
$f(x)$	−1.0526	−1.0050	−1.0005

x	0.001	0.01	0.1
$f(x)$	−0.9995	−0.9950	−0.9524

The estimate of the limit of $f(x)$, as x approaches zero, is -1.00.

5. 5; Proof **7.** -3; Proof **9.** (a) 4 (b) 5 **11.** 16

13. $\sqrt{6} \approx 2.45$ **15.** $-\frac{1}{4}$ **17.** $\frac{1}{2}$ **19.** -1 **21.** 75

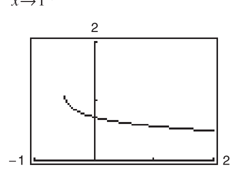
23. 0 **25.** $\sqrt{3}/2$ **27.** $-\frac{1}{2}$ **29.** $\frac{7}{12}$

31. (a)

x	1.1	1.01	1.001	1.0001
$f(x)$	0.5680	0.5764	0.5773	0.5773

$\lim_{x \to 1^+} f(x) \approx 0.5773$

(b)



The graph has a hole at $x = 1$.
$\lim_{x \to 1^+} f(x) \approx 0.5774$

(c) $\sqrt{3}/3$

33. -39.2 m/sec **35.** -1 **37.** 0

39. Limit does not exist. The limit as t approaches 1 from the left is 2 whereas the limit as t approaches 1 from the right is 1.

41. Continuous for all real x

43. Nonremovable discontinuity at each integer
Continuous on $(k, k + 1)$ for all integers k

45. Removable discontinuity at $x = 1$
Continuous on $(-\infty, 1) \cup (1, \infty)$

47. Nonremovable discontinuity at $x = 2$
Continuous on $(-\infty, 2) \cup (2, \infty)$

49. Nonremovable discontinuity at $x = -1$
Continuous on $(-\infty, -1) \cup (-1, \infty)$

51. Nonremovable discontinuity at each even integer
Continuous on $(2k, 2k + 2)$ for all integers k

53. $c = -\frac{1}{2}$ **55.** Proof

57. (a) -4 (b) 4 (c) Limit does not exist.

59. $x = 0$ **61.** $x = 10$ **63.** $-\infty$ **65.** $\frac{1}{3}$

67. $-\infty$ **69.** $-\infty$ **71.** $\frac{4}{5}$ **73.** ∞

75. (a) \$14,117.65 (b) \$80,000.00 (c) \$720,000.00 (d) ∞

P.S. Problem Solving (page 93)

1. (a) Perimeter $\triangle PAO = 1 + \sqrt{(x^2 - 1)^2 + x^2} + \sqrt{x^4 + x^2}$
Perimeter $\triangle PBO = 1 + \sqrt{x^4 + (x - 1)^2} + \sqrt{x^4 + x^2}$

(b)

x	4	2	1
Perimeter $\triangle PAO$	33.0166	9.0777	3.4142
Perimeter $\triangle PBO$	33.7712	9.5952	3.4142
$r(x)$	0.9777	0.9461	1.0000

x	0.1	0.01
Perimeter $\triangle PAO$	2.0955	2.0100
Perimeter $\triangle PBO$	2.0006	2.0000
$r(x)$	1.0475	1.0050

(c) 1

3. (a) Area (hexagon) $= (3\sqrt{3})/2 \approx 2.5981$
Area (circle) $= \pi \approx 3.1416$
Area (circle) $-$ Area (hexagon) ≈ 0.5435

(b) $A_n = (n/2) \sin(2\pi/n)$

(c)

n	6	12	24	48	96
A_n	2.5981	3.0000	3.1058	3.1326	3.1394

(d) 3.1416 or π

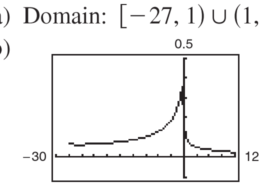
5. (a) $m = -\frac{12}{5}$ (b) $y = \frac{5}{12}x - \frac{169}{12}$

(c) $m_x = \dfrac{-\sqrt{169 - x^2} + 12}{x - 5}$

(d) $\frac{5}{12}$; It is the same as the slope of the tangent line found in (b).

7. (a) Domain: $[-27, 1) \cup (1, \infty)$

(b)

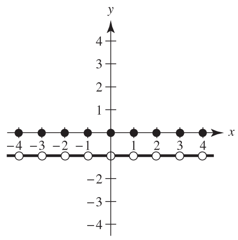


The graph has a hole at $x = 1$.

(c) $\frac{1}{14}$ (d) $\frac{1}{12}$

9. (a) g_1, g_4 (b) g_1 (c) g_1, g_3, g_4

11.



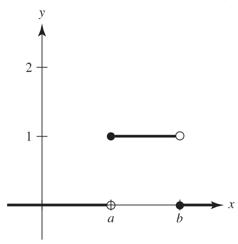
The graph jumps at every integer.

(a) $f(1) = 0,\ f(0) = 0,\ f\left(\frac{1}{2}\right) = -1,\ f(-2.7) = -1$

(b) $\lim_{x\to 1^-} f(x) = -1,\ \lim_{x\to 1^+} f(x) = -1,\ \lim_{x\to 1/2} f(x) = -1$

(c) There is a discontinuity at each integer.

13. (a)



(b) (i) $\lim_{x\to a^+} P_{a,b}(x) = 1$

(ii) $\lim_{x\to a^-} P_{a,b}(x) = 0$

(iii) $\lim_{x\to b^+} P_{a,b}(x) = 0$

(iv) $\lim_{x\to b^-} P_{a,b}(x) = 1$

(c) Continuous for all positive real numbers except a and b

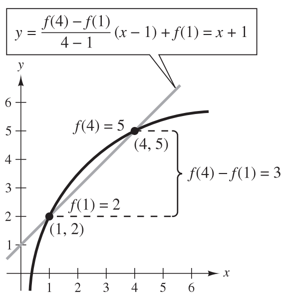
(d) The area under the graph of U and above the x-axis is 1.

Chapter 2

Section 2.1 (page 103)

1. (a) $m_1 = 0, m_2 = 5/2$ (b) $m_1 = -5/2, m_2 = 2$

3.



5. $m = -5$ **7.** $m = 4$

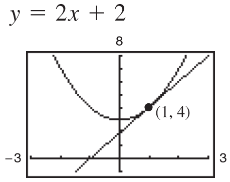
9. $m = 3$ **11.** $f'(x) = 0$ **13.** $f'(x) = -10$ **15.** $h'(s) = \frac{2}{3}$

17. $f'(x) = 2x + 1$ **19.** $f'(x) = 3x^2 - 12$

21. $f'(x) = \dfrac{-1}{(x-1)^2}$ **23.** $f'(x) = \dfrac{1}{2\sqrt{x+4}}$

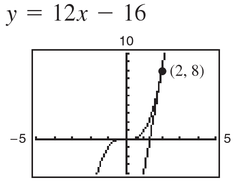
25. (a) Tangent line: $y = 2x + 2$

(b)



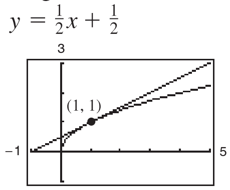
27. (a) Tangent line: $y = 12x - 16$

(b)



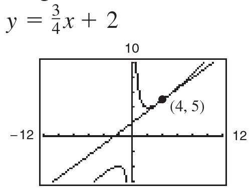
29. (a) Tangent line: $y = \frac{1}{2}x + \frac{1}{2}$

(b)



31. (a) Tangent line: $y = \frac{3}{4}x + 2$

(b)

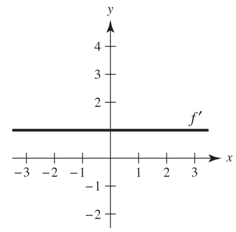


33. $y = 2x - 1$ **35.** $y = 3x - 2;\ y = 3x + 2$

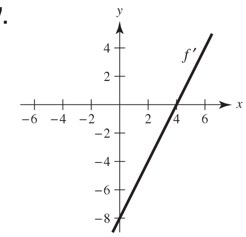
37. $y = -\frac{1}{2}x + \frac{3}{2}$ **39.** b **40.** d **41.** a **42.** c

43. $g(4) = 5;\ g'(4) = -\frac{5}{3}$

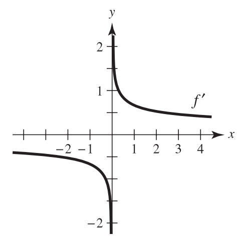
45.



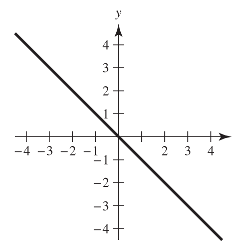
47.



49.



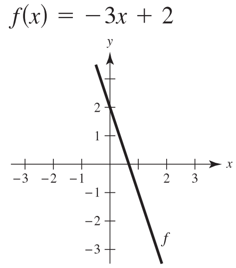
51. Answers will vary.
Sample answer: $y = -x$



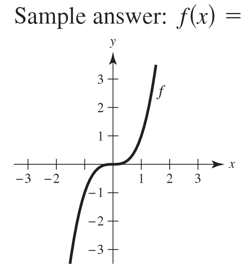
53. $f(x) = 5 - 3x$
$c = 1$

55. $f(x) = -x^2$
$c = 6$

57. $f(x) = -3x + 2$

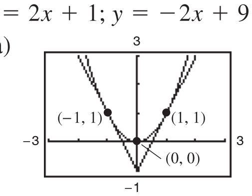


59. Answers will vary.
Sample answer: $f(x) = x^3$



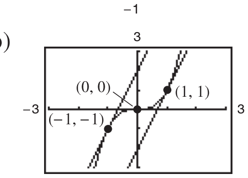
61. $y = 2x + 1;\ y = -2x + 9$

63. (a)



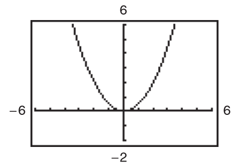
For this function, the slopes of the tangent lines are always distinct for different values of x.

(b)



For this function, the slopes of the tangent lines are sometimes the same.

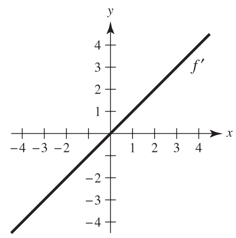
65. (a)



$f'(0) = 0, f'(\frac{1}{2}) = \frac{1}{2}, f'(1) = 1, f'(2) = 2$

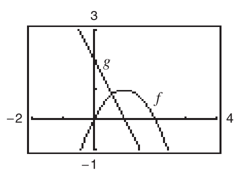
(b) $f'(-\frac{1}{2}) = -\frac{1}{2}, f'(-1) = -1, f'(-2) = -2$

(c)



(d) $f'(x) = x$

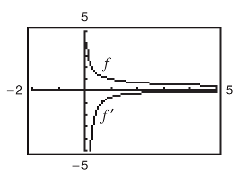
67.



$g(x) \approx f'(x)$

69. $f(2) = 4; f(2.1) = 3.99; f'(2) \approx -0.1$

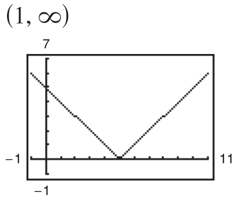
71.



As x approaches infinity, the graph of f approaches a line of slope 0. Thus $f'(x)$ approaches 0.

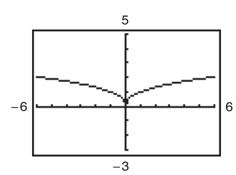
73. 6 **75.** 4 **77.** $g(x)$ is not differentiable at $x = 0$.
79. $f(x)$ is not differentiable at $x = 6$.
81. $h(x)$ is not differentiable at $x = -7$.
83. $(-\infty, 3) \cup (3, \infty)$ **85.** $(-\infty, -4) \cup (-4, \infty)$
87. $(1, \infty)$

89.



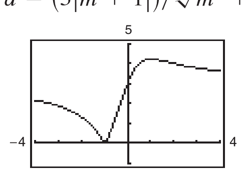
$(-\infty, 5) \cup (5, \infty)$

91.



$(-\infty, 0) \cup (0, \infty)$

93. The derivative from the left is -1 and the derivative from the right is 1, so f is not differentiable at $x = 1$.
95. The derivatives from both the right and the left are 0, so $f'(1) = 0$.
97. f is differentiable at $x = 2$.
99. (a) $d = (3|m + 1|)/\sqrt{m^2 + 1}$
(b)



Not differentiable at $m = -1$

101. False. The slope is $\lim_{\Delta x \to 0} \frac{f(2 + \Delta x) - f(2)}{\Delta x}$.
103. False. For example: $f(x) = |x|$. The derivative from the left and the derivative from the right both exist but are not equal.
105. Proof

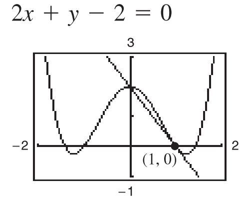
Section 2.2 (page 115)

1. (a) $\frac{1}{2}$ (b) 3 **3.** 0 **5.** $7x^6$ **7.** $-5/x^6$ **9.** $1/(5x^{4/5})$
11. 1 **13.** $-4t + 3$ **15.** $2x + 12x^2$ **17.** $3t^2 + 10t - 3$
19. $\frac{\pi}{2}\cos\theta + \sin\theta$ **21.** $2x + \frac{1}{2}\sin x$ **23.** $-\frac{1}{x^2} - 3\cos x$

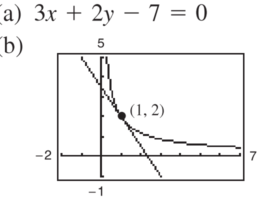
	Function	*Rewrite*	*Derivative*	*Simplify*
25.	$y = \frac{5}{2x^2}$	$y = \frac{5}{2}x^{-2}$	$y' = -5x^{-3}$	$y' = -\frac{5}{x^3}$
27.	$y = \frac{6}{(5x)^3}$	$y = \frac{6}{125}x^{-3}$	$y' = -\frac{18}{125}x^{-4}$	$y' = -\frac{18}{125x^4}$
29.	$y = \frac{\sqrt{x}}{x}$	$y = x^{-1/2}$	$y' = -\frac{1}{2}x^{-3/2}$	$y' = -\frac{1}{2x^{3/2}}$

31. -2 **33.** 0 **35.** 8 **37.** 3 **39.** $2x + 6/x^3$
41. $2t + 12/t^4$ **43.** $8x + 3$ **45.** $(x^3 - 8)/x^3$
47. $3x^2 + 1$ **49.** $\frac{1}{2\sqrt{x}} - \frac{2}{x^{2/3}}$ **51.** $\frac{4}{5s^{1/5}} - \frac{2}{3s^{1/3}}$
53. $\frac{3}{\sqrt{x}} - 5\sin x$

55. (a) $2x + y - 2 = 0$
(b)

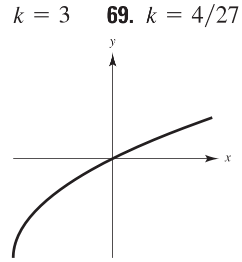


57. (a) $3x + 2y - 7 = 0$
(b)



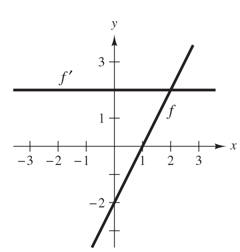
59. $(-1, 2), (0, 3), (1, 2)$ **61.** No horizontal tangents
63. (π, π) **65.** $k = -1, k = -9$
67. $k = 3$ **69.** $k = 4/27$

71.



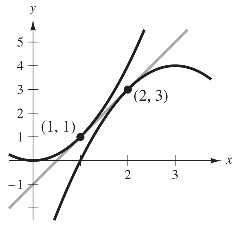
73. $g'(x) = f'(x)$

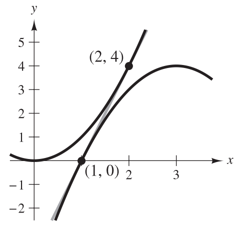
75.



The rate of change of f is constant and therefore f' is a constant function.

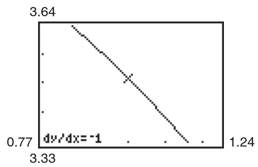
77. $y = 2x - 1$ $\quad$ $y = 4x - 4$





79. $f'(x) = 3 + \cos x \neq 0$ for all x. **81.** $x - 4y + 4 = 0$

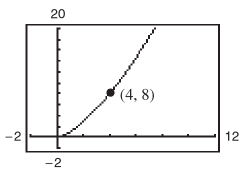
83.



$f'(1)$ appears to be close to -1.

$f'(1) = -1$

85. (a)

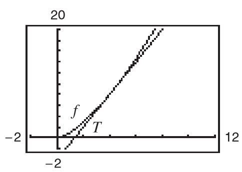


(3.9, 7.7019), $S(x) = 2.981x - 3.924$

(b) $T(x) = 3(x - 4) + 8 = 3x - 4$

The slope (and equation) of the secant line approaches that of the tangent line at (4, 8) as you choose points closer and closer to (4, 8).

(c)



The approximation becomes less accurate.

(d)

Δx	-3	-2	-1	-0.5	-0.1	0
$f(4 + \Delta x)$	1	2.828	5.196	6.548	7.702	8
$T(4 + \Delta x)$	-1	2	5	6.5	7.7	8

Δx	0.1	0.5	1	2	3
$f(4 + \Delta x)$	8.302	9.546	11.180	14.697	18.520
$T(4 + \Delta x)$	8.3	9.5	11	14	17

87. False. Let $f(x) = x$ and $g(x) = x + 1$.

89. False. $dy/dx = 0$ **91.** True

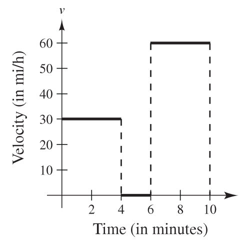
93. Average rate: 4
Instantaneous rates:
$f'(1) = 4; f'(2) = 4$

95. Average rate: $\frac{1}{2}$
Instantaneous rates:
$f'(1) = 1; f'(2) = \frac{1}{4}$

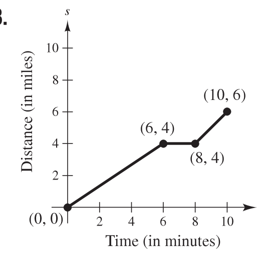
97. (a) $s(t) = -16t^2 + 1362$; $v(t) = -32t$ (b) -48 ft/sec
(c) $s'(1) = -32$ ft/sec; $s'(2) = -64$ ft/sec
(d) $t = \dfrac{\sqrt{1362}}{4} \approx 9.226$ sec (e) -295.242 ft/sec

99. $v(5) = 71$ m/sec; $v(10) = 22$ m/sec

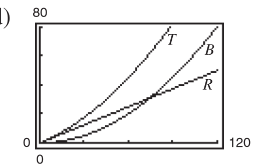
101.



103.



105. (a) $R(v) = 0.417v - 0.02$
(b) $B(v) = 0.0056v^2 + 0.001v + 0.04$
(c) $T(v) = 0.0056v^2 + 0.418v + 0.02$
(d)



(e) $T'(v) = 0.0112v + 0.418$
$T'(40) = 0.866$
$T'(80) = 1.314$
$T'(100) = 1.538$

(f) Stopping distance increases at an increasing rate.

107. $V'(6) = 108 \text{ cm}^3/\text{cm}$ **109.** Proof

111. (a) The rate of change of the number of gallons of gasoline sold when the price is \$2.979
(b) In general, the rate of change when $p = 2.979$ should be negative. As prices go up, sales go down.

113. $y = 2x^2 - 3x + 1$ **115.** $9x + y = 0, 9x + 4y + 27 = 0$

117. $a = \frac{1}{3}, b = -\frac{4}{3}$

119. $f_1(x) = |\sin x|$ is differentiable for all $x \neq n\pi$, n an integer.
$f_2(x) = \sin|x|$ is differentiable for all $x \neq 0$.

Section 2.3 (page 126)

1. $2(2x^3 - 6x^2 + 3x - 6)$ **3.** $(1 - 5t^2)/(2\sqrt{t})$

5. $x^2(3 \cos x - x \sin x)$ **7.** $(1 - x^2)/(x^2 + 1)^2$

9. $(1 - 5x^3)/[2\sqrt{x}(x^3 + 1)^2]$ **11.** $(x \cos x - 2 \sin x)/x^3$

13. $f'(x) = (x^3 + 4x)(6x + 2) + (3x^2 + 2x - 5)(3x^2 + 4)$
$= 15x^4 + 8x^3 + 21x^2 + 16x - 20$
$f'(0) = -20$

15. $f'(x) = \dfrac{x^2 - 6x + 4}{(x - 3)^2}$ $\quad$ $f'(1) = -\dfrac{1}{4}$

17. $f'(x) = \cos x - x \sin x$ $\quad$ $f'\left(\dfrac{\pi}{4}\right) = \dfrac{\sqrt{2}}{8}(4 - \pi)$

	Function	*Rewrite*	*Differentiate*	*Simplify*
19.	$y = \dfrac{x^2 + 3x}{7}$	$y = \dfrac{1}{7}x^2 + \dfrac{3}{7}x$	$y' = \dfrac{2}{7}x + \dfrac{3}{7}$	$y' = \dfrac{2x + 3}{7}$
21.	$y = \dfrac{6}{7x^2}$	$y = \dfrac{6}{7}x^{-2}$	$y' = -\dfrac{12}{7}x^{-3}$	$y' = -\dfrac{12}{7x^3}$
23.	$y = \dfrac{4x^{3/2}}{x}$	$y = 4x^{1/2}$, $x > 0$	$y' = 2x^{-1/2}$	$y' = \dfrac{2}{\sqrt{x}}$, $x > 0$

25. $\dfrac{(x^2 - 1)(-3 - 2x) - (4 - 3x - x^2)(2x)}{(x^2 - 1)^2} = \dfrac{3}{(x + 1)^2}, \ x \neq 1$

27. $1 - 12/(x + 3)^2 = (x^2 + 6x - 3)/(x + 3)^2$

29. $\dfrac{3}{2}x^{-1/2} + \dfrac{1}{2}x^{-3/2} = (3x + 1)/2x^{3/2}$

31. $6s^2(s^3 - 2)$ **33.** $-(2x^2 - 2x + 3)/[x^2(x - 3)^2]$

35. $(6x^2 + 5)(x - 3)(x + 2) + (2x^3 + 5x)(1)(x + 2)$
$\quad + (2x^3 + 5x)(x - 3)(1)$
$\quad = 10x^4 - 8x^3 - 21x^2 - 10x - 30$

37. $\dfrac{(x^2 - c^2)(2x) - (x^2 + c^2)(2x)}{(x^2 - c^2)^2} = -\dfrac{4xc^2}{(x^2 - c^2)^2}$

39. $t(t \cos t + 2 \sin t)$ **41.** $-(t \sin t + \cos t)/t^2$

43. $-1 + \sec^2 x = \tan^2 x$ **45.** $\dfrac{1}{4t^{3/4}} - 6 \csc t \cot t$

47. $\dfrac{-6 \cos^2 x + 6 \sin x - 6 \sin^2 x}{4 \cos^2 x} = \dfrac{3}{2}(-1 + \tan x \sec x - \tan^2 x)$
$\quad = \dfrac{3}{2} \sec x(\tan x - \sec x)$

49. $\csc x \cot x - \cos x = \cos x \cot^2 x$ **51.** $x(x \sec^2 x + 2 \tan x)$

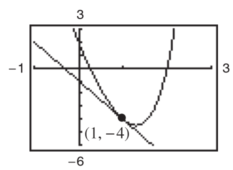
53. $2x \cos x + 2 \sin x - x^2 \sin x + 2x \cos x$
$\quad = 4x \cos x + (2 - x^2) \sin x$

55. $\left(\dfrac{x + 1}{x + 2}\right)(2) + (2x - 5)\left[\dfrac{(x + 2)(1) - (x + 1)(1)}{(x + 2)^2}\right]$
$\quad = \dfrac{2x^2 + 8x - 1}{(x + 2)^2}$

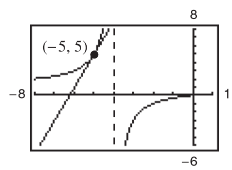
57. $\dfrac{1 - \sin \theta + \theta \cos \theta}{(1 - \sin \theta)^2}$ **59.** $y' = \dfrac{-2 \csc x \cot x}{(1 - \csc x)^2}, \quad -4\sqrt{3}$

61. $h'(t) = \sec t(t \tan t - 1)/t^2, \quad 1/\pi^2$

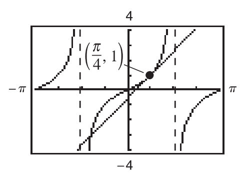
63. (a) $y = -3x - 1$
(b)



65. (a) $y = 4x + 25$
(b)



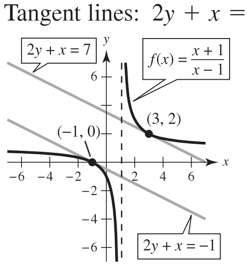
67. (a) $4x - 2y - \pi + 2 = 0$
(b)



69. $2y + x - 4 = 0$

71. $25y - 12x + 16 = 0$ **73.** $(1, 1)$ **75.** $(0, 0), (2, 4)$

77. Tangent lines: $2y + x = 7$; $2y + x = -1$



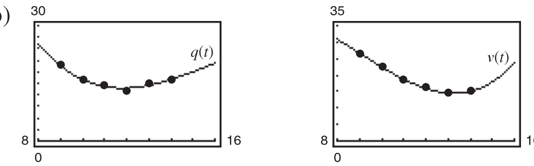
79. $f(x) + 2 = g(x)$ **81.** (a) $p'(1) = 1$ (b) $q'(4) = -1/3$

83. $(18t + 5)/(2\sqrt{t})$ cm²/sec

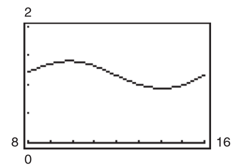
85. (a) −\$38.13 thousand/100 components
(b) −\$10.37 thousand/100 components
(c) −\$3.80 thousand/100 components
The cost decreases with increasing order size.

87. 31.55 bacteria/h **89.** Proof

91. (a) $q(t) = -0.0546t^3 + 2.529t^2 - 36.89t + 186.6$
$v(t) = 0.0796t^3 - 2.162t^2 + 15.32t + 5.9$
(b)



(c) $A = \dfrac{0.0796t^3 - 2.162t^2 + 15.32t + 5.9}{-0.0546t^3 + 2.529t^2 - 36.89t + 186.6}$



A represents the average value (in billions of dollars) per one million personal computers.

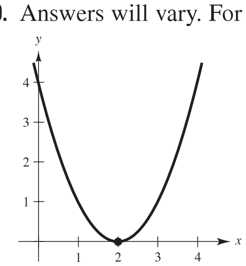
(d) $A'(t)$ represents the rate of change of the average value per one million personal computers for the given year.

93. $12x^2 + 12x - 6$ **95.** $3/\sqrt{x}$ **97.** $2/(x - 1)^3$

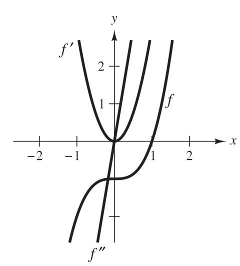
99. $2 \cos x - x \sin x$ **101.** $2x$ **103.** $1/\sqrt{x}$

105. 0 **107.** −10

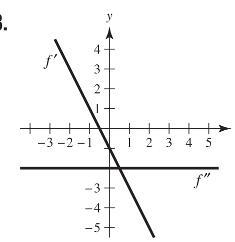
109. Answers will vary. For example: $f(x) = (x - 2)^2$



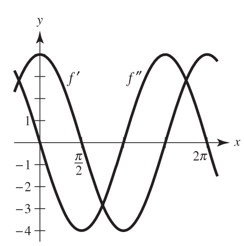
111.



113.



115.



117. $v(3) = 27$ m/sec
$a(3) = -6$ m/sec²
The speed of the object is decreasing.

119.

t	0	1	2	3	4
$s(t)$	0	57.75	99	123.75	132
$v(t)$	66	49.5	33	16.5	0
$a(t)$	−16.5	−16.5	−16.5	−16.5	−16.5

The average velocity on [0, 1] is 57.75, on [1, 2] is 41.25, on [2, 3] is 24.75, and on [3, 4] is 8.25.

121. $f^{(n)}(x) = n(n-1)(n-2)\cdots(2)(1) = n!$

123. (a) $f''(x) = g(x)h''(x) + 2g'(x)h'(x) + g''(x)h(x)$

$f'''(x) = g(x)h'''(x) + 3g'(x)h''(x) + 3g''(x)h'(x) + g'''(x)h(x)$

$f^{(4)}(x) = g(x)h^{(4)}(x) + 4g'(x)h'''(x) + 6g''(x)h''(x) + 4g'''(x)h'(x) + g^{(4)}(x)h(x)$

(b) $f^{(n)}(x) = g(x)h^{(n)}(x) + \dfrac{n!}{1!(n-1)!}g'(x)h^{(n-1)}(x) + \dfrac{n!}{2!(n-2)!}g''(x)h^{(n-2)}(x) + \cdots + \dfrac{n!}{(n-1)!1!}g^{(n-1)}(x)h'(x) + g^{(n)}(x)h(x)$

125. $n = 1$: $f'(x) = x\cos x + \sin x$

$n = 2$: $f'(x) = x^2\cos x + 2x\sin x$

$n = 3$: $f'(x) = x^3\cos x + 3x^2\sin x$

$n = 4$: $f'(x) = x^4\cos x + 4x^3\sin x$

General rule: $f'(x) = x^n\cos x + nx^{(n-1)}\sin x$

127. $y' = -1/x^2$, $y'' = 2/x^3$,

$x^3y'' + 2x^2y' = x^3(2/x^3) + 2x^2(-1/x^2) = 2 - 2 = 0$

129. $y' = 2\cos x$, $y'' = -2\sin x$,

$y'' + y = -2\sin x + 2\sin x + 3 = 3$

131. False. $dy/dx = f(x)g'(x) + g(x)f'(x)$ **133.** True

135. True **137.** $f(x) = 3x^2 - 2x - 1$

139. $f'(x) = 2|x|$; $f''(0)$ does not exist. **141.** Proof

Section 2.4 (page 137)

	$y = f(g(x))$	$u = g(x)$	$y = f(u)$
1.	$y = (5x-8)^4$	$u = 5x - 8$	$y = u^4$
3.	$y = \sqrt{x^3 - 7}$	$u = x^3 - 7$	$y = \sqrt{u}$
5.	$y = \csc^3 x$	$u = \csc x$	$y = u^3$

7. $12(4x-1)^2$ **9.** $-108(4-9x)^3$

11. $\frac{1}{2}(5-t)^{-1/2}(-1) = -1/\left(2\sqrt{5-t}\right)$

13. $\frac{1}{3}(6x^2+1)^{-2/3}(12x) = 4x/\sqrt[3]{(6x^2+1)^2}$

15. $\frac{1}{2}(9-x^2)^{-3/4}(-2x) = -x/\sqrt[4]{(9-x^2)^3}$ **17.** $-1/(x-2)^2$

19. $-2(t-3)^{-3}(1) = -2/(t-3)^3$ **21.** $-1/\left[2\sqrt{(x+2)^3}\right]$

23. $x^2[4(x-2)^3(1)] + (x-2)^4(2x) = 2x(x-2)^3(3x-2)$

25. $x\left(\dfrac{1}{2}\right)(1-x^2)^{-1/2}(-2x) + (1-x^2)^{1/2}(1) = \dfrac{1-2x^2}{\sqrt{1-x^2}}$

27. $\dfrac{(x^2+1)^{1/2}(1) - x(1/2)(x^2+1)^{-1/2}(2x)}{x^2+1} = \dfrac{1}{\sqrt{(x^2+1)^3}}$

29. $\dfrac{-2(x+5)(x^2+10x-2)}{(x^2+2)^3}$ **31.** $\dfrac{-9(1-2v)^2}{(v+1)^4}$

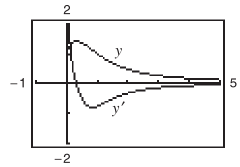
33. $2((x^2+3)^5 + x)(5(x^2+3)^4(2x) + 1)$

$= 20x(x^2+3)^9 + 2(x^2+3)^5 + 20x^2(x^2+3)^4 + 2x$

35. $\dfrac{1}{2}(2 + (2 + x^{1/2})^{1/2})^{-1/2}\left(\dfrac{1}{2}(2 + x^{1/2})^{-1/2}\right)\left(\dfrac{1}{2}x^{-1/2}\right)$

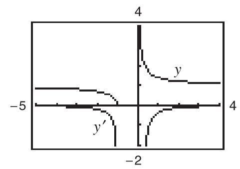
$= \dfrac{1}{8\sqrt{x}\left(\sqrt{2+\sqrt{x}}\right)\left(\sqrt{2+\sqrt{2+\sqrt{x}}}\right)}$

37. $(1 - 3x^2 - 4x^{3/2})/[2\sqrt{x}(x^2+1)^2]$



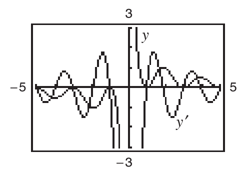
The zero of y' corresponds to the point on the graph of the function where the tangent line is horizontal.

39. $-\dfrac{\sqrt{\dfrac{x+1}{x}}}{2x(x+1)}$



y' has no zeros.

41. $-[\pi x\sin(\pi x) + \cos(\pi x) + 1]/x^2$



The zeros of y' correspond to the points on the graph of the function where the tangent lines are horizontal.

43. (a) 1 (b) 2; The slope of $\sin ax$ at the origin is a.

45. $-4\sin 4x$ **47.** $15\sec^2 3x$ **49.** $2\pi^2 x\cos(\pi x)^2$

51. $2\cos 4x$ **53.** $(-1 - \cos^2 x)/\sin^3 x$

55. $8\sec^2 x\tan x$ **57.** $10\tan 5\theta\sec^2 5\theta$

59. $\sin 2\theta\cos 2\theta = \dfrac{1}{2}\sin 4\theta$

61. $\dfrac{6\pi\sin(\pi t - 1)}{\cos^3(\pi t - 1)}$ **63.** $\dfrac{1}{2\sqrt{x}} + 2x\cos(2x)^2$

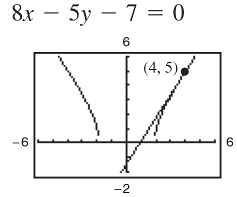
65. $2\sec^2 2x\cos(\tan 2x)$

67. $s'(t) = \dfrac{t+3}{\sqrt{t^2+6t-2}}, \dfrac{6}{5}$ **69.** $f'(x) = \dfrac{-15x^2}{(x^3-2)^2}, -\dfrac{3}{5}$

71. $f'(t) = \dfrac{-5}{(t-1)^2}, -5$ **73.** $y' = -12\sec^3 4x\tan 4x, 0$

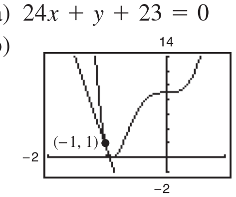
75. (a) $8x - 5y - 7 = 0$

(b)



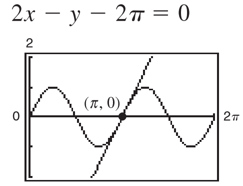
77. (a) $24x + y + 23 = 0$

(b)



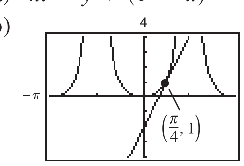
79. (a) $2x - y - 2\pi = 0$

(b)



81. (a) $4x - y + (1 - \pi) = 0$

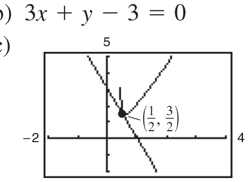
(b)



83. (a) $g'(1/2) = -3$

(b) $3x + y - 3 = 0$

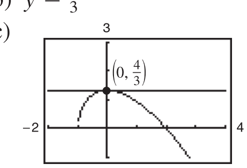
(c)



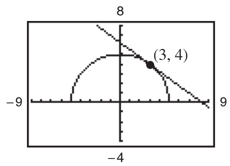
85. (a) $s'(0) = 0$

(b) $y = \frac{4}{3}$

(c)



87. $3x + 4y - 25 = 0$



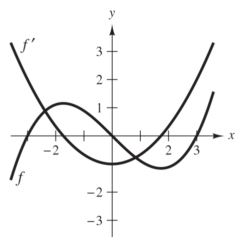
89. $\left(\frac{\pi}{6}, \frac{3\sqrt{3}}{2}\right), \left(\frac{5\pi}{6}, -\frac{3\sqrt{3}}{2}\right), \left(\frac{3\pi}{2}, 0\right)$ **91.** $2940(2 - 7x)^2$

93. $\frac{2}{(x - 6)^3}$

95. $2(\cos x^2 - 2x^2 \sin x^2)$ **97.** $h''(x) = 18x + 6, 24$

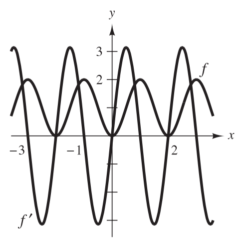
99. $f''(x) = -4x^2 \cos(x^2) - 2\sin(x^2), 0$

101.



The zeros of f' correspond to the points where the graph of f has horizontal tangents.

103.



The zeros of f' correspond to the points where the graph of f has horizontal tangents.

105. The rate of change of g is three times as fast as the rate of change of f.

107. (a) $g'(x) = f'(x)$ (b) $h'(x) = 2f'(x)$
(c) $r'(x) = -3f'(-3x)$ (d) $s'(x) = f'(x + 2)$

x	-2	-1	0	1	2	3
$f'(x)$	4	$\frac{2}{3}$	$-\frac{1}{3}$	-1	-2	-4
$g'(x)$	4	$\frac{2}{3}$	$-\frac{1}{3}$	-1	-2	-4
$h'(x)$	8	$\frac{4}{3}$	$-\frac{2}{3}$	-2	-4	-8
$r'(x)$		12	1			
$s'(x)$	$-\frac{1}{3}$	-1	-2	-4		

109. (a) $\frac{1}{2}$
(b) $s'(5)$ does not exist because g is not differentiable at 6.

111. (a) 1.461 (b) -1.016

113. 0.2 rad, 1.45 rad/sec **115.** 0.04224 cm/sec

117. (a) $x = -1.637t^3 + 19.31t^2 - 0.5t - 1$
(b) $\frac{dC}{dt} = -294.66t^2 + 2317.2t - 30$
(c) Because x, the number of units produced in t hours, is not a linear function, and therefore the cost with respect to time t is not linear.

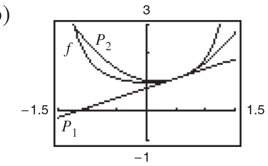
119. (a) Yes, if $f(x + p) = f(x)$ for all x, then $f'(x + p) = f'(x)$, which shows that f' is periodic as well.
(b) Yes, if $g(x) = f(2x)$, then $g'(x) = 2f'(2x)$. Because f' is periodic, so is g'.

121. (a) 0
(b) $f'(x) = 2\sec x \cdot \sec x \tan x = 2\sec^2 x \tan x$
$g'(x) = 2\tan x \sec^2 x = 2\sec^2 x \tan x$
$f'(x) = g'(x)$

123. Proof **125.** $f'(x) = 2x\left(\frac{x^2 - 9}{|x^2 - 9|}\right), x \neq \pm 3$

127. $f'(x) = \cos x \sin x / |\sin x|, x \neq k\pi$

129. (a) $P_1(x) = 2/3(x - \pi/6) + 2/\sqrt{3}$
$P_2(x) = 5/(3\sqrt{3})(x - \pi/6)^2 + 2/3(x - \pi/6) + 2/\sqrt{3}$
(b)



(c) P_2
(d) The accuracy worsens as you move away from $x = \pi/6$.

131. False. If $f(x) = \sin^2 2x$, then $f'(x) = 2(\sin 2x)(2\cos 2x)$.

133. Putnam Problem A1, 1967

Section 2.5 (page 146)

1. $-x/y$ **3.** $-\sqrt{y/x}$ **5.** $(y - 3x^2)/(2y - x)$

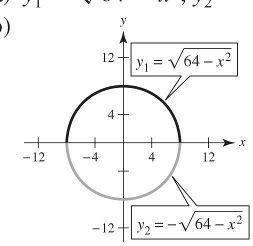
7. $(1 - 3x^2y^3)/(3x^3y^2 - 1)$

9. $(6xy - 3x^2 - 2y^2)/(4xy - 3x^2)$

11. $\cos x/[4\sin(2y)]$ **13.** $(\cos x - \tan y - 1)/(x\sec^2 y)$

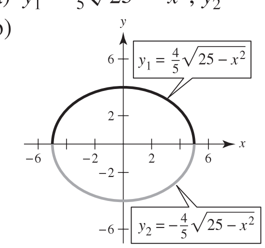
15. $[y\cos(xy)]/[1 - x\cos(xy)]$

17. (a) $y_1 = \sqrt{64 - x^2}$; $y_2 = -\sqrt{64 - x^2}$
(b)



(c) $y' = \mp\frac{x}{\sqrt{64 - x^2}} = -\frac{x}{y}$
(d) $y' = -\frac{x}{y}$

19. (a) $y_1 = \frac{4}{5}\sqrt{25 - x^2}$; $y_2 = -\frac{4}{5}\sqrt{25 - x^2}$
(b)



(c) $y' = \mp\frac{4x}{5\sqrt{25 - x^2}} = -\frac{16x}{25y}$
(d) $y' = -\frac{16x}{25y}$

21. $-\frac{y}{x}, -\frac{1}{6}$ **23.** $\frac{98x}{y(x^2 + 49)^2}$, Undefined **25.** $-\sqrt[3]{\frac{y}{x}}, -\frac{1}{2}$

27. $-\sin^2(x + y)$ or $-\frac{x^2}{x^2 + 1}, 0$ **29.** $-\frac{1}{2}$ **31.** 0

33. $y = -x + 7$ **35.** $y = -x + 2$

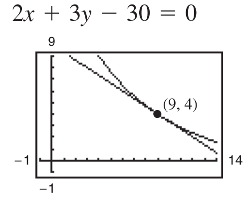
37. $y = \sqrt{3}x/6 + 8\sqrt{3}/3$ **39.** $y = -\frac{2}{11}x + \frac{30}{11}$

41. (a) $y = -2x + 4$ (b) Answers will vary.

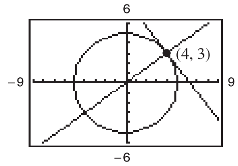
43. $\cos^2 y, -\frac{\pi}{2} < y < \frac{\pi}{2}, \frac{1}{1 + x^2}$ **45.** $-4/y^3$

47. $-36/y^3$ **49.** $(3x)/(4y)$

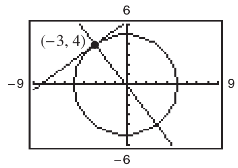
51. $2x + 3y - 30 = 0$



53. At $(4, 3)$:
Tangent line: $4x + 3y - 25 = 0$
Normal line: $3x - 4y = 0$



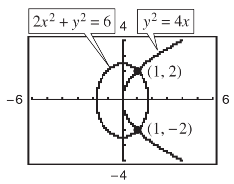
At $(-3, 4)$:
Tangent line: $3x - 4y + 25 = 0$
Normal line: $4x + 3y = 0$



55. $x^2 + y^2 = r^2 \Rightarrow y' = -x/y \Rightarrow y/x =$ slope of normal line. Then for (x_0, y_0) on the circle, $x_0 \neq 0$, an equation of the normal line is $y = (y_0/x_0)x$, which passes through the origin. If $x_0 = 0$, the normal line is vertical and passes through the origin.

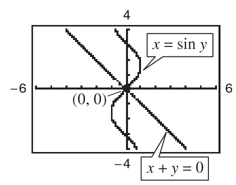
57. Horizontal tangents: $(-4, 0), (-4, 10)$
Vertical tangents: $(0, 5), (-8, 5)$

59.



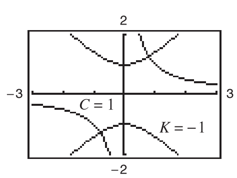
At $(1, 2)$:
Slope of ellipse: -1
Slope of parabola: 1
At $(1, -2)$:
Slope of ellipse: 1
Slope of parabola: -1

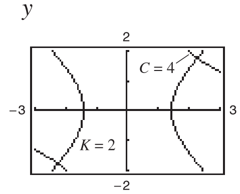
61.



At $(0, 0)$:
Slope of line: -1
Slope of sine curve: 1

63. Derivatives: $\dfrac{dy}{dx} = -\dfrac{y}{x}, \dfrac{dy}{dx} = \dfrac{x}{y}$



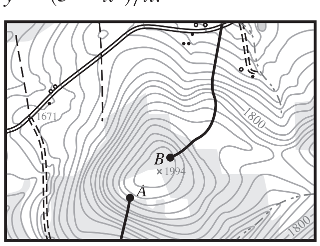


65. (a) $\dfrac{dy}{dx} = \dfrac{3x^3}{y}$ (b) $y\dfrac{dy}{dt} = 3x^3\dfrac{dx}{dt}$

67. (a) $\dfrac{dy}{dx} = \dfrac{-3\cos \pi x}{\sin \pi y}$ (b) $-\sin \pi y\left(\dfrac{dy}{dt}\right) = 3\cos \pi x\left(\dfrac{dx}{dt}\right)$

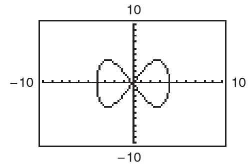
69. Answers will vary. In the explicit form of a function, the variable is explicitly written as a function of x. In an implicit equation, the function is only implied by an equation. An example of an implicit function is $x^2 + xy = 5$. In explicit form it would be $y = (5 - x^2)/x$.

71.

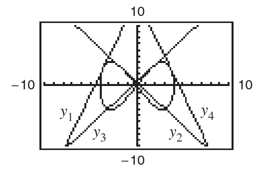


Use starting point B.

73. (a)



(b)



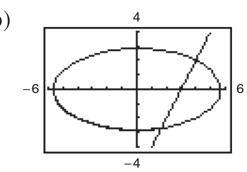
(c) $\left(\dfrac{8\sqrt{7}}{7}, 5\right)$

$$y_1 = \tfrac{1}{3}\left[\left(\sqrt{7} + 7\right)x + \left(8\sqrt{7} + 23\right)\right]$$
$$y_2 = -\tfrac{1}{3}\left[\left(-\sqrt{7} + 7\right)x - \left(23 - 8\sqrt{7}\right)\right]$$
$$y_3 = -\tfrac{1}{3}\left[\left(\sqrt{7} - 7\right)x - \left(23 - 8\sqrt{7}\right)\right]$$
$$y_4 = -\tfrac{1}{3}\left[\left(\sqrt{7} + 7\right)x - \left(8\sqrt{7} + 23\right)\right]$$

75. Proof **77.** $(6, -8), (-6, 8)$

79. $y = -\dfrac{\sqrt{3}}{2}x + 2\sqrt{3}, y = \dfrac{\sqrt{3}}{2}x - 2\sqrt{3}$

81. (a) $y = 2x - 6$
(b)



(c) $\left(\frac{28}{17}, -\frac{46}{17}\right)$

Section 2.6 (page 154)

1. (a) $\frac{3}{4}$ (b) 20 **3.** (a) $-\frac{5}{8}$ (b) $\frac{3}{2}$
5. (a) -8 cm/sec (b) 0 cm/sec (c) 8 cm/sec
7. (a) 8 cm/sec (b) 4 cm/sec (c) 2 cm/sec
9. In a linear function, if x changes at a constant rate, so does y. However, unless $a = 1$, y does not change at the same rate as x.
11. $(4x^3 + 6x)/\sqrt{x^4 + 3x^2 + 1}$
13. (a) 64π cm²/min (b) 256π cm²/min
15. (a) Proof
(b) When $\theta = \dfrac{\pi}{6}, \dfrac{dA}{dt} = \dfrac{\sqrt{3}}{8}s^2$. When $\theta = \dfrac{\pi}{3}, \dfrac{dA}{dt} = \dfrac{1}{8}s^2$.
(c) If s and $d\theta/dt$ are constant, dA/dt is proportional to $\cos\theta$.
17. (a) $2/(9\pi)$ cm/min (b) $1/(18\pi)$ cm/min
19. (a) 144 cm²/sec (b) 720 cm²/sec **21.** $8/(405\pi)$ ft/min
23. (a) 12.5% (b) $\frac{1}{144}$ m/min
25. (a) $-\frac{7}{12}$ ft/sec; $-\frac{3}{2}$ ft/sec; $-\frac{48}{7}$ ft/sec
(b) $\frac{527}{24}$ ft²/sec (c) $\frac{1}{12}$ rad/sec
27. Rate of vertical change: $\frac{1}{5}$ m/sec
Rate of horizontal change: $-\sqrt{3}/15$ m/sec
29. (a) -750 mi/h (b) 30 min
31. $-50/\sqrt{85} \approx -5.42$ ft/sec **33.** (a) $\frac{25}{3}$ ft/sec (b) $\frac{10}{3}$ ft/sec
35. (a) 12 sec (b) $\frac{1}{2}\sqrt{3}$ m (c) $\left(\sqrt{5}\pi\right)/120$ m/sec

37. Evaporation rate proportional to $S \Rightarrow \dfrac{dV}{dt} = k(4\pi r^2)$

$V = \left(\dfrac{4}{3}\right)\pi r^3 \Rightarrow \dfrac{dV}{dt} = 4\pi r^2 \dfrac{dr}{dt}$. So $k = \dfrac{dr}{dt}$.

39. 0.6 ohm/sec **41.** $\dfrac{dv}{dt} = \dfrac{16r}{v}\sec^2\theta \dfrac{d\theta}{dt}, \dfrac{d\theta}{dt} = \dfrac{v}{16r}\cos^2\theta \dfrac{dv}{dt}$

43. $\dfrac{2\sqrt{21}}{525} \approx 0.017$ rad/sec

45. (a) $\dfrac{200\pi}{3}$ ft/sec (b) 200π ft/sec (c) About 427.43π ft/sec

47. About 84.9797 mi/h

49. (a) $\dfrac{dy}{dt} = 3\dfrac{dx}{dt}$ means that y changes three times as fast as x changes.
(b) y changes slowly when $x \approx 0$ or $x \approx L$. y changes more rapidly when x is near the middle of the interval.

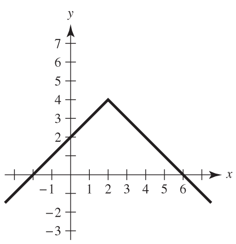
51. -18.432 ft/sec^2 **53.** About -97.96 m/sec

Review Exercises for Chapter 2 (page 158)

1. $f'(x) = 2x - 4$ **3.** $f'(x) = -2/(x-1)^2$

5. f is differentiable at all $x \neq 3$.

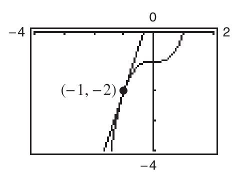
7.



(a) Yes
(b) No, because the derivatives from the left and right are not equal.

9. $-\frac{3}{2}$

11. (a) $y = 3x + 1$
(b)

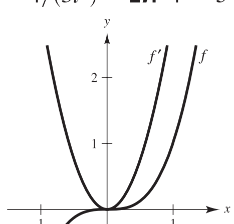


13. 8

15. 0 **17.** $8x^7$ **19.** $52t^3$ **21.** $3x^2 - 22x$ **23.** $\dfrac{3}{\sqrt{x}} + \dfrac{1}{\sqrt[3]{x^2}}$

25. $-4/(3t^3)$ **27.** $4 - 5\cos\theta$ **29.** $-3\sin\theta - (\cos\theta)/4$

31.

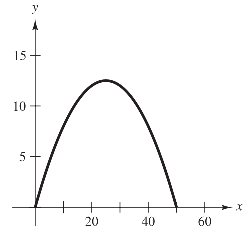


$f' > 0$ where the slopes of tangent lines to the graph of f are positive.

33. (a) 50 vibrations/sec/lb
(b) 33.33 vibrations/sec/lb

35. 1354.24 ft or 412.77 m

37. (a)



(b) 50
(c) $x = 25$
(d) $y' = 1 - 0.04x$

x	0	10	25	30	50
y'	1	0.6	0	-0.2	-1

(e) $y'(25) = 0$

39. (a) $x'(t) = 2t - 3$ (b) $(-\infty, 1.5)$ (c) $x = -\frac{1}{4}$ (d) 1

41. $4(5x^3 - 15x^2 - 11x - 8)$ **43.** $\sqrt{x}\cos x + \sin x/(2\sqrt{x})$

45. $-(x^2+1)/(x^2-1)^2$ **47.** $(8x)/(9-4x^2)^2$

49. $\dfrac{4x^3\cos x + x^4 \sin x}{\cos^2 x}$ **51.** $3x^2\sec x\tan x + 6x\sec x$

53. $-x\sin x$ **55.** $y = 4x - 3$ **57.** $y = 0$

59. $v(4) = 20$ m/sec; $a(4) = -8$ m/sec^2

61. $-48t$ **63.** $\frac{225}{4}\sqrt{x}$ **65.** $6\sec^2\theta\tan\theta$

67. $y'' + y = -(2\sin x + 3\cos x) + (2\sin x + 3\cos x) = 0$

69. $\dfrac{2(x+5)(-x^2-10x+3)}{(x^2+3)^3}$

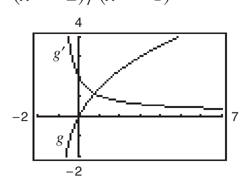
71. $s(s^2-1)^{3/2}(8s^3 - 3s + 25)$

73. $-45\sin(9x+1)$ **75.** $\frac{1}{2}(1 - \cos 2x) = \sin^2 x$

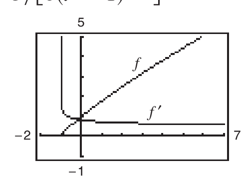
77. $\sin^{1/2}x\cos x - \sin^{5/2}x\cos x = \cos^3 x\sqrt{\sin x}$

79. $\dfrac{(x+2)(\pi\cos\pi x) - \sin\pi x}{(x+2)^2}$ **81.** -2 **83.** 0

85. $(x+2)/(x+1)^{3/2}$ **87.** $5/[6(t+1)^{1/6}]$

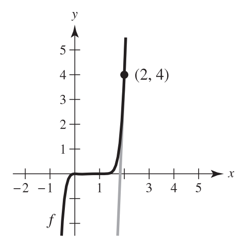


g' is not equal to zero for any x.

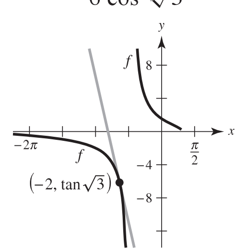


f' has no zeros.

89. (a) $f'(2) = 24$ (b) $y = 24t - 44$
(c)



91. (a) $f'(-2) = -\dfrac{1}{2\sqrt{3}\cos^2\sqrt{3}} \approx -11.1983$
(b) $y = -\dfrac{\sqrt{3}(x+2)}{6\cos^2\sqrt{3}} + \tan\sqrt{3}$
(c)



93. $14 - 4\cos 2x$ **95.** $2\csc^2 x \cot x$

97. $[8(2t + 1)]/(1 - t)^4$

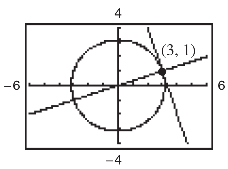
99. $18\sec^2 3\theta \tan 3\theta + \sin(\theta - 1)$

101. (a) $-18.667°/\text{h}$ (b) $-7.284°/\text{h}$
(c) $-3.240°/\text{h}$ (d) $-0.747°/\text{h}$

103. $-\dfrac{2x + 3y}{3(x + y^2)}$ **105.** $\dfrac{\sqrt{y}(2\sqrt{x} - \sqrt{y})}{\sqrt{x}(\sqrt{x} + 8\sqrt{y})} = \dfrac{2x - 9y}{9x - 32y}$

107. $\dfrac{y \sin x + \sin y}{\cos x - x \cos y}$

109. Tangent line: $3x + y - 10 = 0$
Normal line: $x - 3y = 0$



111. (a) $2\sqrt{2}$ units/sec (b) 4 units/sec (c) 8 units/sec

113. $\frac{2}{25}$ m/min **115.** -38.34 m/sec

P.S. Problem Solving (page 161)

1. (a) $r = \frac{1}{2}; x^2 + \left(y - \frac{1}{2}\right)^2 = \frac{1}{4}$
(b) Center: $\left(0, \frac{5}{4}\right); x^2 + \left(y - \frac{5}{4}\right)^2 = 1$

3. (a) $P_1(x) = 1$ (b) $P_2(x) = 1 - \frac{1}{2}x^2$
(c)

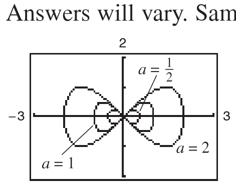
x	-1.0	-0.1	-0.001	0	0.001
$\cos x$	0.5403	0.9950	1.000	1	1.000
$P_2(x)$	0.5	0.995	1.000	1	1.000

x	0.1	1.0
$\cos x$	0.9950	0.5403
$P_2(x)$	0.995	0.5

$P_2(x)$ is a good approximation of $f(x) = \cos x$ when x is very close to 0.
(d) $P_3(x) = x - \frac{1}{6}x^3$

5. $p(x) = 2x^3 + 4x^2 - 5$

7. (a) Graph $\begin{cases} y_1 = \dfrac{1}{a}\sqrt{x^2(a^2 - x^2)} \\ y_2 = -\dfrac{1}{a}\sqrt{x^2(a^2 - x^2)} \end{cases}$ as separate equations.
(b) Answers will vary. Sample answer:



The intercepts will always be $(0, 0)$, $(a, 0)$, and $(-a, 0)$, and the maximum and minimum y-values appear to be $\pm\frac{1}{2}a$.
(c) $\left(\dfrac{a\sqrt{2}}{2}, \dfrac{a}{2}\right), \left(\dfrac{a\sqrt{2}}{2}, -\dfrac{a}{2}\right), \left(-\dfrac{a\sqrt{2}}{2}, \dfrac{a}{2}\right), \left(-\dfrac{a\sqrt{2}}{2}, -\dfrac{a}{2}\right)$

9. (a) When the man is 90 ft from the light, the tip of his shadow is $112\frac{1}{2}$ ft from the light. The tip of the child's shadow is $111\frac{1}{9}$ ft from the light, so the man's shadow extends $1\frac{7}{18}$ ft beyond the child's shadow.
(b) When the man is 60 ft from the light, the tip of his shadow is 75 ft from the light. The tip of the child's shadow is $77\frac{7}{9}$ ft from the light, so the child's shadow extends $2\frac{7}{9}$ ft beyond the man's shadow.
(c) $d = 80$ ft
(d) Let x be the distance of the man from the light and let s be the distance from the light to the tip of the shadow.
If $0 < x < 80$, $ds/dt = -50/9$.
If $x > 80$, $ds/dt = -25/4$.
There is a discontinuity at $x = 80$.

11. Proof. The graph of L is a line passing through the origin $(0, 0)$.

13. (a)

$z°$	0.1	0.01	0.0001
$\dfrac{\sin z}{z}$	0.0174532837	0.0174532924	0.0174532925

(b) $\pi/180$ (c) $(\pi/180)\cos z$
(d) $S(90) = 1$, $C(180) = -1$; $(\pi/180)C(z)$
(e) Answers will vary.

15. (a) j would be the rate of change of acceleration.
(b) $j = 0$. Acceleration is constant, so there is no change in acceleration.
(c) a: position function, d: velocity function,
b: acceleration function, c: jerk function

Chapter 3

Section 3.1 (page 169)

1. $f'(0) = 0$ **3.** $f'(2) = 0$ **5.** $f'(-2)$ is undefined.

7. 2, absolute maximum (and relative maximum)

9. 1, absolute maximum (and relative maximum);
2, absolute minimum (and relative minimum);
3, absolute maximum (and relative maximum)

11. $x = 0, x = 2$ **13.** $t = 8/3$ **15.** $x = \pi/3, \pi, 5\pi/3$

17. Minimum: $(2, 1)$
Maximum: $(-1, 4)$

19. Minimum: $(1, -1)$
Maximum: $(4, 8)$

21. Minimum: $\left(-1, -\frac{5}{2}\right)$
Maximum: $(2, 2)$

23. Minimum: $(0, 0)$
Maximum: $(-1, 5)$

25. Minimum: $(0, 0)$
Maxima: $\left(-1, \frac{1}{4}\right)$ and $\left(1, \frac{1}{4}\right)$

27. Minimum: $(1, -1)$
Maximum: $\left(0, -\frac{1}{2}\right)$

29. Minimum: $(-1, -1)$
Maximum: $(3, 3)$

31. Minimum value is -2 for $-2 \le x < -1$.
Maximum: $(2, 2)$

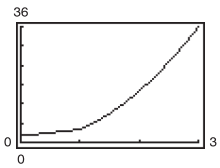
33. Minimum: $\left(1/6, \sqrt{3}/2\right)$
Maximum: $(0, 1)$

35. Minimum: $(\pi, -3)$
Maxima: $(0, 3)$ and $(2\pi, 3)$

37. (a) Minimum: $(0, -3)$;
Maximum: $(2, 1)$
(b) Minimum: $(0, -3)$
(c) Maximum: $(2, 1)$
(d) No extrema

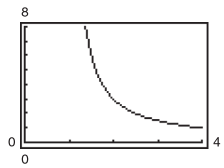
39. (a) Minimum: $(1, -1)$;
Maximum: $(-1, 3)$
(b) Maximum: $(3, 3)$
(c) Minimum: $(1, -1)$
(d) Minimum: $(1, -1)$

41.



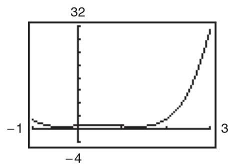
Minimum: $(0, 2)$
Maximum: $(3, 36)$

43.



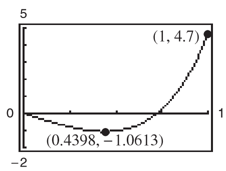
Minimum: $(4, 1)$

45.



Minima: $\left(\frac{-\sqrt{3}+1}{2}, \frac{3}{4}\right)$ and $\left(\frac{\sqrt{3}+1}{2}, \frac{3}{4}\right)$
Maximum: $(3, 31)$

47. (a)

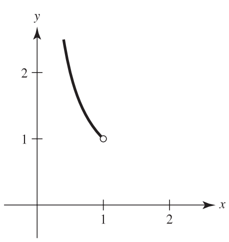


(b) Minimum: $(0.4398, -1.0613)$

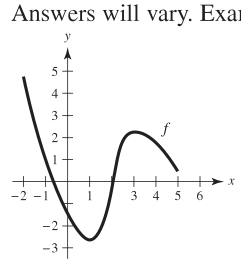
49. Maximum: $\left|f''\left(\sqrt[3]{-10+\sqrt{108}}\right)\right| = f''\left(\sqrt{3}-1\right) \approx 1.47$

51. Maximum: $\left|f^{(4)}(0)\right| = \frac{56}{81}$

53. Answers will vary. Let $f(x) = 1/x$. f is continuous on $(0, 1)$ but does not have a maximum or minimum.



55. Answers will vary. Example:



57. (a) Yes (b) No **59.** (a) No (b) Yes

61. Maximum: $P(12) = 72$; No. P is decreasing for $I > 12$.

63. $\theta = \operatorname{arcsec}\sqrt{3} \approx 0.9553$ rad

65. True **67.** True **69.** Proof **71.** Putnam Problem B3, 2004

Section 3.2 (page 176)

1. $f(-1) = f(1) = 1$; f is not continuous on $[-1, 1]$.

3. $f(0) = f(2) = 0$; f is not differentiable on $(0, 2)$.

5. $(2, 0), (-1, 0)$; $f'\left(\frac{1}{2}\right) = 0$ **7.** $(0, 0), (-4, 0)$; $f'\left(-\frac{8}{3}\right) = 0$

9. $f'(-1) = 0$ **11.** $f'\left(\frac{3}{2}\right) = 0$

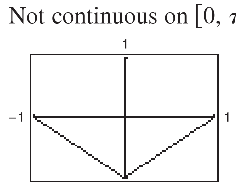
13. $f'\left(\frac{6-\sqrt{3}}{3}\right) = 0$; $f'\left(\frac{6+\sqrt{3}}{3}\right) = 0$

15. Not differentiable at $x = 0$ **17.** $f'\left(-2+\sqrt{5}\right) = 0$

19. $f'(\pi/2) = 0$; $f'(3\pi/2) = 0$ **21.** $f'(0.249) \approx 0$

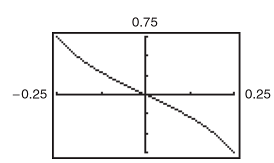
23. Not continuous on $[0, \pi]$

25.



Rolle's Theorem does not apply.

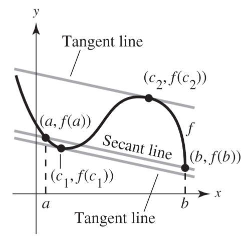
27.



Rolle's Theorem does not apply.

29. (a) $f(1) = f(2) = 38$
(b) Velocity $= 0$ for some t in $(1, 2)$; $t = \frac{3}{2}$ sec

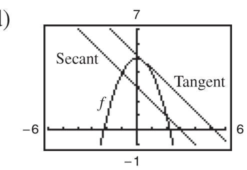
31.



33. The function is not continuous on $[0, 6]$.

35. The function is not continuous on $[0, 6]$.

37. (a) Secant line: $x + y - 3 = 0$ (b) $c = \frac{1}{2}$
(c) Tangent line: $4x + 4y - 21 = 0$
(d)

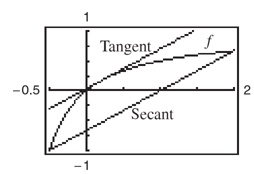


39. $f'(-1/2) = -1$ **41.** $f'\left(1/\sqrt{3}\right) = 3, f'\left(-1/\sqrt{3}\right) = 3$

43. $f'(8/27) = 1$ **45.** f is not differentiable at $x = -\frac{1}{2}$.

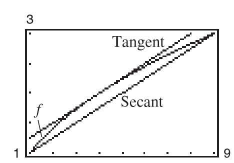
47. $f'(\pi/2) = 0$

49. (a)–(c)



(b) $y = \frac{2}{3}(x - 1)$
(c) $y = \frac{1}{3}\left(2x + 5 - 2\sqrt{6}\right)$

51. (a)–(c)



(b) $y = \frac{1}{4}x + \frac{3}{4}$
(c) $y = \frac{1}{4}x + 1$

53. (a) -14.7 m/sec (b) 1.5 sec

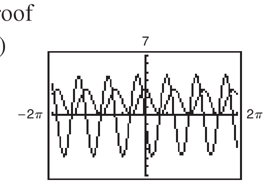
55. No. Let $f(x) = x^2$ on $[-1, 2]$.

57. No. $f(x)$ is not continuous on $[0, 1]$. So it does not satisfy the hypothesis of Rolle's Theorem.

59. By the Mean Value Theorem, there is a time when the speed of the plane must equal the average speed of 454.5 miles/hour. The speed was 400 miles/hour when the plane was accelerating to 454.5 miles/hour and decelerating from 454.5 miles/hour.

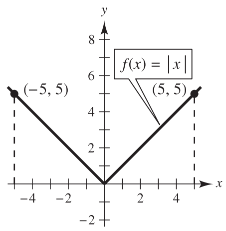
61. Proof

63. (a)



(b) Yes; yes
(c) Because $f(-1) = f(1) = 0$, Rolle's Theorem applies on $[-1, 1]$. Because $f(1) = 0$ and $f(2) = 3$, Rolle's Theorem does not apply on $[1, 2]$.
(d) $\lim_{x\to 3^-} f'(x) = 0$; $\lim_{x\to 3^+} f'(x) = 0$

65.



67. Proof **69.** Proof

71. $a = 6, b = 1, c = 2$ **73.** $f(x) = 5$ **75.** $f(x) = x^2 - 1$

77. False. f is not continuous on $[-1, 1]$. **79.** True

81–89. Proofs

Section 3.3 (page 186)

1. (a) $(0, 6)$ (b) $(6, 8)$

3. Increasing on $(3, \infty)$; Decreasing on $(-\infty, 3)$

5. Increasing on $(-\infty, -2)$ and $(2, \infty)$; Decreasing on $(-2, 2)$

7. Increasing on $(-\infty, -1)$; Decreasing on $(-1, \infty)$

9. Increasing on $(1, \infty)$; Decreasing on $(-\infty, 1)$

11. Increasing on $(-2\sqrt{2}, 2\sqrt{2})$
Decreasing on $(-4, -2\sqrt{2})$ and $(2\sqrt{2}, 4)$

13. Increasing on $(0, \pi/2)$ and $(3\pi/2, 2\pi)$;
Decreasing on $(\pi/2, 3\pi/2)$

15. Increasing on $(0, 7\pi/6)$ and $(11\pi/6, 2\pi)$;
Decreasing on $(7\pi/6, 11\pi/6)$

17. (a) Critical number: $x = 2$
(b) Increasing on $(2, \infty)$; Decreasing on $(-\infty, 2)$
(c) Relative minimum: $(2, -4)$

19. (a) Critical number: $x = 1$
(b) Increasing on $(-\infty, 1)$; Decreasing on $(1, \infty)$
(c) Relative maximum: $(1, 5)$

21. (a) Critical numbers: $x = -2, 1$
(b) Increasing on $(-\infty, -2)$ and $(1, \infty)$; Decreasing on $(-2, 1)$
(c) Relative maximum: $(-2, 20)$; Relative minimum: $(1, -7)$

23. (a) Critical numbers: $x = -\frac{5}{3}, 1$
(b) Increasing on $\left(-\infty, -\frac{5}{3}\right), (1, \infty)$
Decreasing on $\left(-\frac{5}{3}, 1\right)$
(c) Relative maximum: $\left(-\frac{5}{3}, \frac{256}{27}\right)$
Relative minimum: $(1, 0)$

25. (a) Critical numbers: $x = \pm 1$
(b) Increasing on $(-\infty, -1)$ and $(1, \infty)$; Decreasing on $(-1, 1)$
(c) Relative maximum: $\left(-1, \frac{4}{5}\right)$; Relative minimum: $\left(1, -\frac{4}{5}\right)$

27. (a) Critical number: $x = 0$
(b) Increasing on $(-\infty, \infty)$
(c) No relative extrema

29. (a) Critical number: $x = -2$
(b) Increasing on $(-2, \infty)$; Decreasing on $(-\infty, -2)$
(c) Relative minimum: $(-2, 0)$

31. (a) Critical number: $x = 5$
(b) Increasing on $(-\infty, 5)$; Decreasing on $(5, \infty)$
(c) Relative maximum: $(5, 5)$

33. (a) Critical numbers: $x = \pm\sqrt{2}/2$; Discontinuity: $x = 0$
(b) Increasing on $(-\infty, -\sqrt{2}/2)$ and $(\sqrt{2}/2, \infty)$
Decreasing on $(-\sqrt{2}/2, 0)$ and $(0, \sqrt{2}/2)$
(c) Relative maximum: $(-\sqrt{2}/2, -2\sqrt{2})$
Relative minimum: $(\sqrt{2}/2, 2\sqrt{2})$

35. (a) Critical number: $x = 0$; Discontinuities: $x = \pm 3$
(b) Increasing on $(-\infty, -3)$ and $(-3, 0)$
Decreasing on $(0, 3)$ and $(3, \infty)$
(c) Relative maximum: $(0, 0)$

37. (a) Critical numbers: $x = -3, 1$; Discontinuity: $x = -1$
(b) Increasing on $(-\infty, -3)$ and $(1, \infty)$
Decreasing on $(-3, -1)$ and $(-1, 1)$
(c) Relative maximum: $(-3, -8)$; Relative minimum: $(1, 0)$

39. (a) Critical number: $x = 0$
(b) Increasing on $(-\infty, 0)$; Decreasing on $(0, \infty)$
(c) No relative extrema

41. (a) Critical number: $x = 1$
(b) Increasing on $(-\infty, 1)$; Decreasing on $(1, \infty)$
(c) Relative maximum: $(1, 4)$

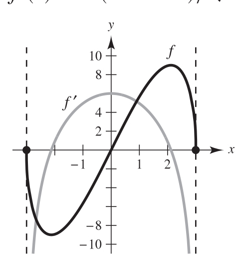
43. (a) Critical numbers: $x = \pi/6, 5\pi/6$
Increasing on $(0, \pi/6), (5\pi/6, 2\pi)$
Decreasing on $(\pi/6, 5\pi/6)$
(b) Relative maximum: $\left(\pi/6, (\pi + 6\sqrt{3})/12\right)$
Relative minimum: $\left(5\pi/6, (5\pi - 6\sqrt{3})/12\right)$

45. (a) Critical numbers: $x = \pi/4, 5\pi/4$
Increasing on $(0, \pi/4), (5\pi/4, 2\pi)$
Decreasing on $(\pi/4, 5\pi/4)$
(b) Relative maximum: $(\pi/4, \sqrt{2})$
Relative minimum: $(5\pi/4, -\sqrt{2})$

47. (a) Critical numbers:
$x = \pi/4, \pi/2, 3\pi/4, \pi, 5\pi/4, 3\pi/2, 7\pi/4$
Increasing on $(\pi/4, \pi/2), (3\pi/4, \pi), (5\pi/4, 3\pi/2), (7\pi/4, 2\pi)$
Decreasing on $(0, \pi/4), (\pi/2, 3\pi/4), (\pi, 5\pi/4), (3\pi/2, 7\pi/4)$
(b) Relative maxima: $(\pi/2, 1), (\pi, 1), (3\pi/2, 1)$
Relative minima: $(\pi/4, 0), (3\pi/4, 0), (5\pi/4, 0), (7\pi/4, 0)$

49. (a) Critical numbers: $\pi/2, 7\pi/6, 3\pi/2, 11\pi/6$
Increasing on $\left(0, \frac{\pi}{2}\right), \left(\frac{7\pi}{6}, \frac{3\pi}{2}\right), \left(\frac{11\pi}{6}, 2\pi\right)$
Decreasing on $\left(\frac{\pi}{2}, \frac{7\pi}{6}\right), \left(\frac{3\pi}{2}, \frac{11\pi}{6}\right)$
(b) Relative maxima: $\left(\frac{\pi}{2}, 2\right), \left(\frac{3\pi}{2}, 0\right)$
Relative minima: $\left(\frac{7\pi}{6}, -\frac{1}{4}\right), \left(\frac{11\pi}{6}, -\frac{1}{4}\right)$

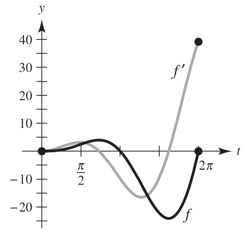
51. (a) $f'(x) = 2(9 - 2x^2)/\sqrt{9 - x^2}$
(b)



(c) Critical numbers: $x = \pm 3\sqrt{2}/2$
(d) $f' > 0$ on $(-3\sqrt{2}/2, 3\sqrt{2}/2)$
$f' < 0$ on $(-3, -3\sqrt{2}/2), (3\sqrt{2}/2, 3)$

53. (a) $f'(t) = t(t\cos t + 2\sin t)$

(b)



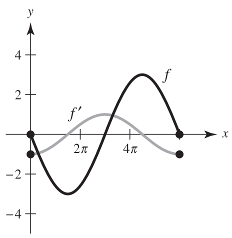
(c) Critical numbers: $t = 2.2889, 5.0870$

(d) $f' > 0$ on $(0, 2.2889), (5.0870, 2\pi)$

$f' < 0$ on $(2.2889, 5.0870)$

55. (a) $f'(x) = -\cos(x/3)$

(b)

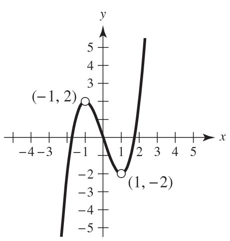


(c) Critical numbers: $x = 3\pi/2, 9\pi/2$

(d) $f' > 0$ on $\left(\frac{3\pi}{2}, \frac{9\pi}{2}\right)$

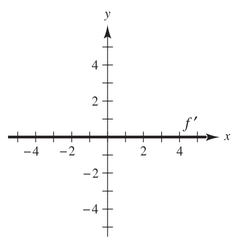
$f' < 0$ on $\left(0, \frac{3\pi}{2}\right), \left(\frac{9\pi}{2}, 6\pi\right)$

57. $f(x)$ is symmetric with respect to the origin.
Zeros: $(0, 0), (\pm\sqrt{3}, 0)$

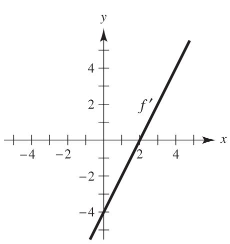


$g(x)$ is continuous on $(-\infty, \infty)$ and $f(x)$ has holes at $x = 1$ and $x = -1$.

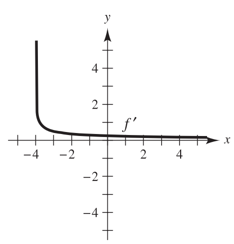
59.



61.



63.



65. (a) Increasing on $(2, \infty)$; Decreasing on $(-\infty, 2)$

(b) Relative minimum: $x = 2$

67. (a) Increasing on $(-\infty, -1)$ and $(0, 1)$;
Decreasing on $(-1, 0)$ and $(1, \infty)$

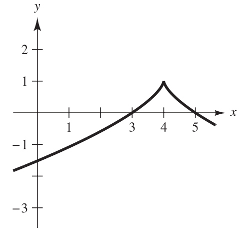
(b) Relative maxima: $x = -1$ and $x = 1$
Relative minimum: $x = 0$

69. (a) Critical numbers: $x = -1, x = 1, x = 2$

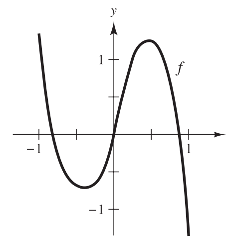
(b) Relative maximum at $x = 1$, relative minimum at $x = 2$, and neither at $x = -1$

71. $g'(0) < 0$ **73.** $g'(-6) < 0$ **75.** $g'(0) > 0$

77. Answers will vary. Sample answer:



79. (a)



(b) Critical numbers: $x \approx -0.40$ and $x \approx 0.48$

(c) Relative maximum: $(0.48, 1.25)$
Relative minimum: $(-0.40, 0.75)$

81. (a) $s'(t) = 9.8(\sin\theta)t$; speed $= |9.8(\sin\theta)t|$

(b)

θ	0	$\pi/4$	$\pi/3$	$\pi/2$	$2\pi/3$	$3\pi/4$	π
$s'(t)$	0	$4.9\sqrt{2}t$	$4.9\sqrt{3}t$	$9.8t$	$4.9\sqrt{3}t$	$4.9\sqrt{2}t$	0

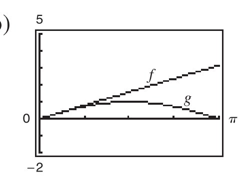
The speed is maximum at $\theta = \pi/2$.

83. (a)

x	0.5	1	1.5	2	2.5	3
$f(x)$	0.5	1	1.5	2	2.5	3
$g(x)$	0.48	0.84	1.00	0.91	0.60	0.14

$f(x) > g(x)$

(b)



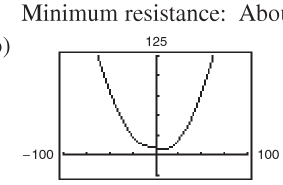
$f(x) > g(x)$

(c) Proof

85. $r = 2R/3$

87. (a) $\dfrac{dR}{dT} = \dfrac{0.004T^3 - 4}{2\sqrt{0.001T^4 - 4T + 100}}$
Critical number: $T = 10$
Minimum resistance: About 8.3666 ohms

(b)



Minimum resistance: About 8.3666 ohms

89. (a) $v(t) = 6 - 2t$ (b) $(0, 3)$ (c) $(3, \infty)$ (d) $t = 3$

91. (a) $v(t) = 3t^2 - 10t + 4$
(b) $\left(0, \left(5 - \sqrt{13}\right)/3\right)$ and $\left(\left(5 + \sqrt{13}\right)/3, \infty\right)$
(c) $\left(\dfrac{5 - \sqrt{13}}{3}, \dfrac{5 + \sqrt{13}}{3}\right)$ (d) $t = \dfrac{5 \pm \sqrt{13}}{3}$

93. Answers will vary.

95. (a) Minimum degree: 3
(b) $a_3(0)^3 + a_2(0)^2 + a_1(0) + a_0 = 0$
$a_3(2)^3 + a_2(2)^2 + a_1(2) + a_0 = 2$
$3a_3(0)^2 + 2a_2(0) + a_1 = 0$
$3a_3(2)^2 + 2a_2(2) + a_1 = 0$
(c) $f(x) = -\frac{1}{2}x^3 + \frac{3}{2}x^2$

97. (a) Minimum degree: 4
(b) $a_4(0)^4 + a_3(0)^3 + a_2(0)^2 + a_1(0) + a_0 = 0$
$a_4(2)^4 + a_3(2)^3 + a_2(2)^2 + a_1(2) + a_0 = 4$
$a_4(4)^4 + a_3(4)^3 + a_2(4)^2 + a_1(4) + a_0 = 0$
$4a_4(0)^3 + 3a_3(0)^2 + 2a_2(0) + a_1 = 0$
$4a_4(2)^3 + 3a_3(2)^2 + 2a_2(2) + a_1 = 0$
$4a_4(4)^3 + 3a_3(4)^2 + 2a_2(4) + a_1 = 0$
(c) $f(x) = \frac{1}{4}x^4 - 2x^3 + 4x^2$

99. True **101.** False. Let $f(x) = x^3$.

103. False. Let $f(x) = x^3$. There is a critical number at $x = 0$, but not a relative extremum.

105–107. Proofs

Section 3.4 (page 195)

1. $f' > 0, f'' > 0$ **3.** $f' < 0, f'' < 0$

5. Concave upward: $(-\infty, \infty)$

7. Concave upward: $(-\infty, 1)$; Concave downward: $(1, \infty)$

9. Concave upward: $(-\infty, 2)$; Concave downward: $(2, \infty)$

11. Concave upward: $(-\infty, -2), (2, \infty)$
Concave downward: $(-2, 2)$

13. Concave upward: $(-\infty, -1), (1, \infty)$
Concave downward: $(-1, 1)$

15. Concave upward: $(-2, 2)$
Concave downward: $(-\infty, -2), (2, \infty)$

17. Concave upward: $(-\pi/2, 0)$; Concave downward: $(0, \pi/2)$

19. Points of inflection: $(-2, -8), (0, 0)$
Concave upward: $(-\infty, -2), (0, \infty)$
Concave downward: $(-2, 0)$

21. Point of inflection: $(2, 8)$; Concave downward: $(-\infty, 2)$
Concave upward: $(2, \infty)$

23. Points of inflection: $\left(\pm 2\sqrt{3}/3, -20/9\right)$
Concave upward: $\left(-\infty, -2\sqrt{3}/3\right), \left(2\sqrt{3}/3, \infty\right)$
Concave downward: $\left(-2\sqrt{3}/3, 2\sqrt{3}/3\right)$

25. Points of inflection: $(2, -16), (4, 0)$
Concave upward: $(-\infty, 2), (4, \infty)$; Concave downward: $(2, 4)$

27. Concave upward: $(-3, \infty)$

29. Points of inflection: $\left(-\sqrt{3}/3, 3\right), \left(\sqrt{3}/3, 3\right)$
Concave upward: $\left(-\infty, -\sqrt{3}/3\right), \left(\sqrt{3}/3, \infty\right)$
Concave downward: $\left(-\sqrt{3}/3, \sqrt{3}/3\right)$

31. Point of inflection: $(2\pi, 0)$
Concave upward: $(2\pi, 4\pi)$; Concave downward: $(0, 2\pi)$

33. Concave upward: $(0, \pi), (2\pi, 3\pi)$
Concave downward: $(\pi, 2\pi), (3\pi, 4\pi)$

35. Points of inflection: $(\pi, 0), (1.823, 1.452), (4.46, -1.452)$
Concave upward: $(1.823, \pi), (4.46, 2\pi)$
Concave downward: $(0, 1.823), (\pi, 4.46)$

37. Relative minimum: $(5, 0)$ **39.** Relative maximum: $(3, 9)$

41. Relative maximum: $(0, 3)$; Relative minimum: $(2, -1)$

43. Relative minimum: $(3, -25)$

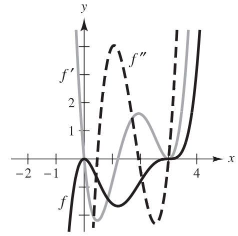
45. Relative maximum: $(2.4, 268.74)$; Relative minimum: $(0, 0)$

47. Relative minimum: $(0, -3)$

49. Relative maximum: $(-2, -4)$; Relative minimum: $(2, 4)$

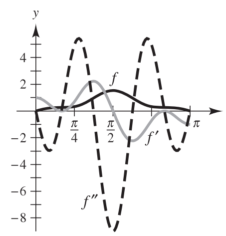
51. No relative extrema, because f is nonincreasing.

53. (a) $f'(x) = 0.2x(x - 3)^2(5x - 6)$
$f''(x) = 0.4(x - 3)(10x^2 - 24x + 9)$
(b) Relative maximum: $(0, 0)$
Relative minimum: $(1.2, -1.6796)$
Points of inflection: $(0.4652, -0.7048)$, $(1.9348, -0.9048)$, $(3, 0)$
(c)



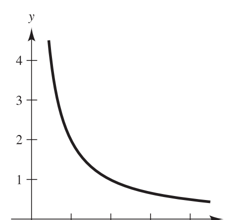
f is increasing when f' is positive, and decreasing when f' is negative. f is concave upward when f'' is positive, and concave downward when f'' is negative.

55. (a) $f'(x) = \cos x - \cos 3x + \cos 5x$
$f''(x) = -\sin x + 3 \sin 3x - 5 \sin 5x$
(b) Relative maximum: $(\pi/2, 1.53333)$
Points of inflection: $(\pi/6, 0.2667), (1.1731, 0.9637)$, $(1.9685, 0.9637), (5\pi/6, 0.2667)$
(c)

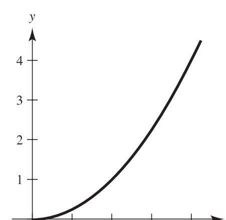


f is increasing when f' is positive, and decreasing when f' is negative. f is concave upward when f'' is positive, and concave downward when f'' is negative.

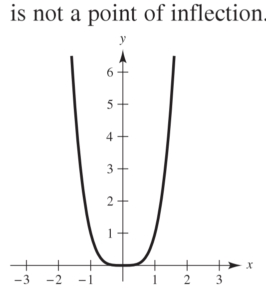
57. (a)



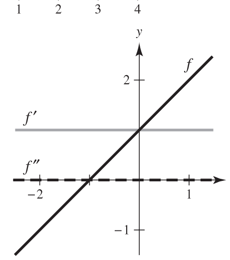
(b)



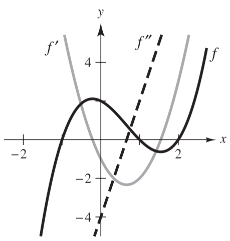
59. Answers will vary. Example: $f(x) = x^4$; $f''(0) = 0$, but $(0, 0)$ is not a point of inflection.



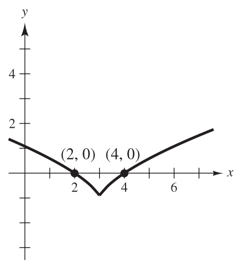
61.



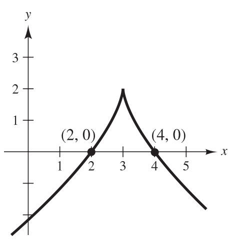
63.



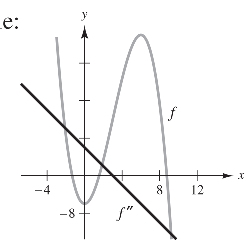
65.



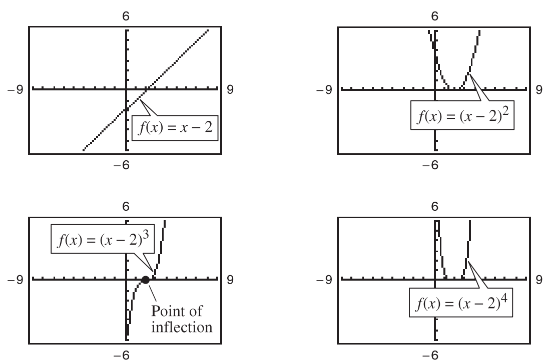
67.



69. Example:



71. (a) $f(x) = (x - 2)^n$ has a point of inflection at $(2, 0)$ if n is odd and $n \geq 3$.



(b) Proof

73. $f(x) = \frac{1}{2}x^3 - 6x^2 + \frac{45}{2}x - 24$

75. (a) $f(x) = \frac{1}{32}x^3 + \frac{3}{16}x^2$ (b) Two miles from touchdown

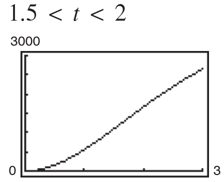
77. $x = \left(\dfrac{15 - \sqrt{33}}{16}\right)L \approx 0.578L$ **79.** $x = 100$ units

81. (a)

t	0.5	1	1.5	2	2.5	3
S	151.5	555.6	1097.6	1666.7	2193.0	2647.1

$1.5 < t < 2$

(b)



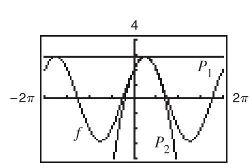
$t \approx 1.5$

(c) About 1.633 yr

83. $P_1(x) = 2\sqrt{2}$

$P_2(x) = 2\sqrt{2} - \sqrt{2}(x - \pi/4)^2$

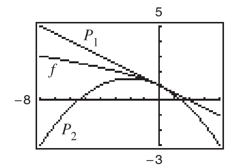
The values of f, P_1, and P_2 and their first derivatives are equal when $x = \pi/4$. The approximations worsen as you move away from $x = \pi/4$.



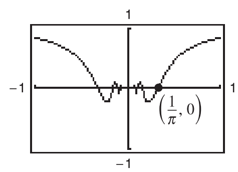
85. $P_1(x) = 1 - x/2$

$P_2(x) = 1 - x/2 - x^2/8$

The values of f, P_1, and P_2 and their first derivatives are equal when $x = 0$. The approximations worsen as you move away from $x = 0$.



87.



89. Proof **91.** True

93. False. f is concave upward at $x = c$ if $f''(c) > 0$. **95.** Proof

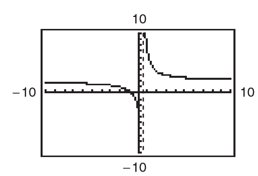
Section 3.5 (page 205)

1. f **2.** c **3.** d **4.** a **5.** b **6.** e

7.

x	10^0	10^1	10^2	10^3
$f(x)$	7	2.2632	2.0251	2.0025

x	10^4	10^5	10^6
$f(x)$	2.0003	2.0000	2.0000

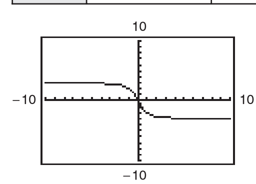


$$\lim_{x\to\infty} \frac{4x + 3}{2x - 1} = 2$$

9.

x	10^0	10^1	10^2	10^3
$f(x)$	−2	−2.9814	−2.9998	−3.0000

x	10^4	10^5	10^6
$f(x)$	−3.0000	−3.0000	−3.0000

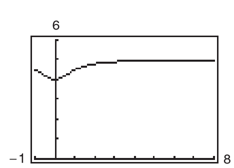


$$\lim_{x\to\infty} \frac{-6x}{\sqrt{4x^2 + 5}} = -3$$

11.

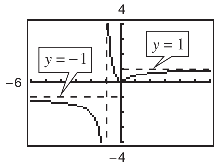
x	10^0	10^1	10^2	10^3
$f(x)$	4.5000	4.9901	4.9999	5.0000

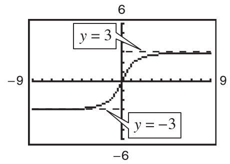
x	10^4	10^5	10^6
$f(x)$	5.0000	5.0000	5.0000



$$\lim_{x\to\infty} \left(5 - \frac{1}{x^2 + 1}\right) = 5$$

13. (a) ∞ (b) 5 (c) 0 **15.** (a) 0 (b) 1 (c) ∞
17. (a) 0 (b) $-\frac{2}{3}$ (c) $-\infty$ **19.** 4 **21.** $\frac{2}{3}$ **23.** 0
25. $-\infty$ **27.** -1 **29.** -2 **31.** $\frac{1}{2}$ **33.** ∞
35. 0 **37.** 0

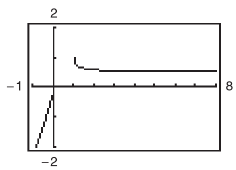
39.


41.


43. 1 **45.** 0 **47.** $\frac{1}{6}$

49.

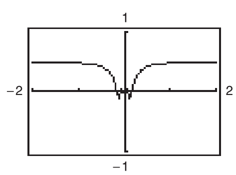
x	10^0	10^1	10^2	10^3	10^4	10^5	10^6
$f(x)$	1.000	0.513	0.501	0.500	0.500	0.500	0.500



$\lim_{x\to\infty}\left[x - \sqrt{x(x-1)}\right] = \frac{1}{2}$

51.

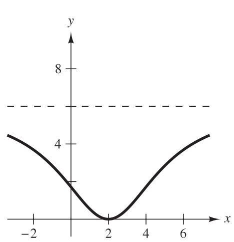
x	10^0	10^1	10^2	10^3	10^4	10^5	10^6
$f(x)$	0.479	0.500	0.500	0.500	0.500	0.500	0.500



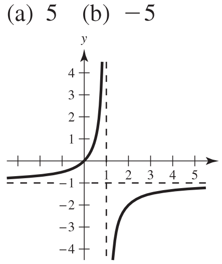
The graph has a hole at $x = 0$.
$\lim_{x\to\infty} x \sin\frac{1}{2x} = \frac{1}{2}$

53. As x becomes large, $f(x)$ approaches 4.

55. Answers will vary. Example: let $f(x) = \dfrac{-6}{0.1(x-2)^2 + 1} + 6$.



57. (a) 5 (b) -5

59.


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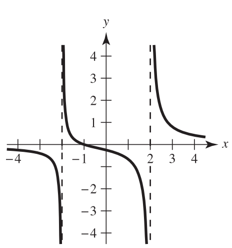

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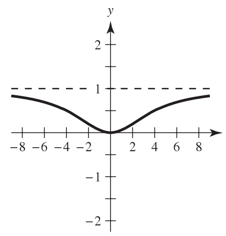

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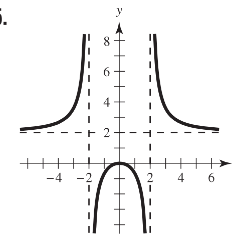

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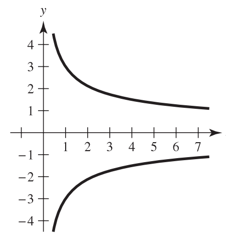

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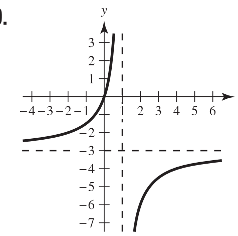

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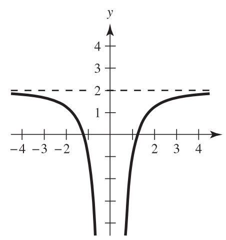

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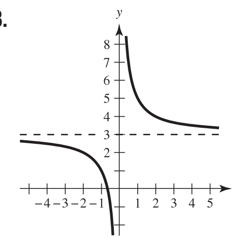

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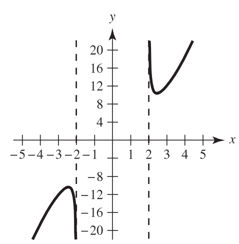

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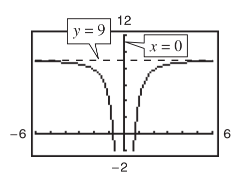

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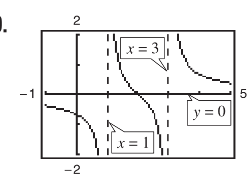

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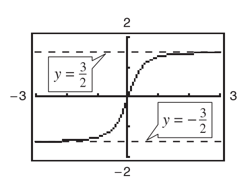

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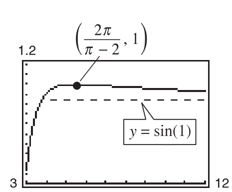
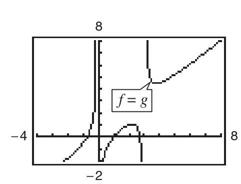
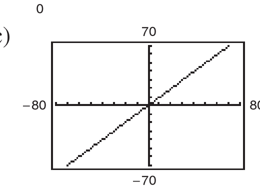


85. (a)


(b) Proof

(c)


The slant asymptote $y = x$

87. 100% **89.** $\lim_{t\to\infty} N(t) = +\infty$; $\lim_{t\to\infty} E(t) = c$